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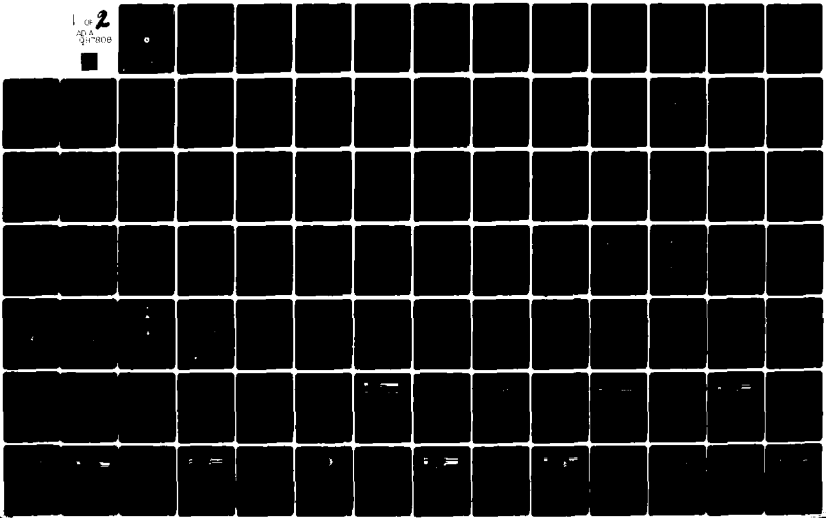

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EVALUATION OF SDRC DAMPING ANALYSIS

M. D. Noll
U. S. COAST GUARD

with Damping Analysis
by
Structural Dynamics Research Corporation

FINAL REPORT

JANUARY 1980

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16. Abstract

Structural Dynamic Research Cooperation (SDRC) obtained FM magnetic tape from Teledyne Engineering Services containing specified stress records from three Great Lakes vessels: M/V STEWART J. CORT, M/V ROGER BLOUGH, and S/S EDWARD L. RYERSON. Under USCG contract, SDRC was tasked to determine the damping value of the intervals from these records. The intervals chosen represented periods of springing aboard the vessels.

To determine a relative "feel" for the accuracy of the results obtained by SDRC, LT M. D. NOLL, U.S.C.G. Office of Research and Development, performed an evaluation which consisted of a manual calculation of damping and logarithmic decrement from associated quick-looks and a comparison of these with the results obtained by SDRC. Although the comparison does not show complete agreement, this evaluation does conclude that the SDRC analysis did provide reasonable values and may be valuable for future data reduction and analysis.

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EVALUATION OF SDRC DAMPING ANALYSIS

LT M. D. NOLL

U. S. COAST GUARD
RESEARCH AND DEVELOPMENT

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Introduction

During the past two decades, vessel designs have made significant advances which include increased speeds, reduced hydrodynamic resistance, increased pay loads, and increased longitudinal strength. Great Lake ore carriers have also made significant advances but with a special restriction not felt by ocean-going vessels. The Great Lake ore carrier has a maximum beam (B) and hull depth (D) restriction due to physical limitations of the locks at Sault Ste. Marie, Michigan. However, these Great Lake vessels have increased in length to 1000 feet with an appropriate lengthening of the locks by the U.S. Army Corps of Engineers.

Of interest to the U.S. Coast Guard, the American Bureau of Shipping (ABS) and the shipping industry is the longitudinal strength required of these long, shallow ore ships exceeding 750' in length. As shown in Table 1, the L/D ratio has increased from 18.7 to 22.23 giving rise to more flexible ships. These vessels experience a phenomenon known as "springing", a 2-node vertical oscillation of the hull structure, caused by the wave encounter frequency matching and being in phase with the natural frequency of the hull girder. The addition of springing to the still water and wave induced bending moments may cause large cross-sectional dynamic stress fluctuations. These increased stresses and associated longitudinal bending moments must be accounted for in the search for a safe and effective longitudinal strength standard.

One illusive parameter in the study of springing has been the damping factor (ζ). System damping is a combination of the hydrodynamic damping, structural damping, cargo damping and the out-of-phase seaway damping. In order to determine the damping factor for Great Lake ore carriers in general, stress recordings from strain gauges mounted on three ore carriers have been analyzed in the effort to determine the damping factor. The three vessels used were the M/V STEWART J. CORT, M/V ROGER BLOUGH, and S/S EDWARD L. RYERSON. Table 1 lists the comparative characteristics for these vessels. The stress records from these vessels have been recorded and analyzed with the results available in References 1,2, and 3.

TABLE 1 - Vessel Characteristics

	CORT	BLOUGH	RYERSON
LOA (ft)	998.0	833	730
B (ft)	105.164	105	75
L/B	9.49	7.9	9.73
D (ft)	44.9	39.2	39.0
L/D	22.23	21.3	18.7
B/D	2.34	2.68	1.92
G.T.(tons)	32930	22041	12170
N.T.(tons)	29918	14114	7637
H.P.	14000	14000	9000
Yr. Built	1971	1971	1960
Owner	Beth. Steel	U.S. Steel	Inland Steel

Background

In June 1977, Teledyne Engineering Services (formerly Teledyne Materials Research) of Waltham, Massachusetts was contracted by the U. S. Coast Guard to transfer specified intervals of analog tape stress records from these three vessels onto FM magnetic tape. These intervals were chosen to represent high, medium and low springing stresses experienced by the vessels during both the loaded and ballasted condition. The intervals were chosen by the Coast Guard from the computer tables listed in Reference 1,2, and 3. Tables 2-4 (Columns 1, 2, 3 and 14) provide the listing of the intervals and maximum peak-to-through (P-T) stresses which Teledyne was contracted to copy over onto magnetic tape. Tape Number and interval number correspond to computer tables in these references. (For example: Table 2, Tape No. "18", Interval No. "20" corresponds to same Tape Number and Interval Number in Reference 1, Appendix B, page 48).

Upon receipt of the FM magnetic tapes and the quick-looks from Teledyne, Structural Dynamics Research Corporation (SDRC) of Cincinnati, Ohio was contracted to determine the range of damping values for these specific intervals. SDRC took possession of the tapes and the quick-looks for their damping analysis.

SDRC Determination

For each specific interval, SDRC supplied a table of estimated roots and an associated damping power spectra. The tables of modal parameters and the data plots are connected by a FREQRESP-BODE number located at the top of the table and at the bottom of the plot (i.e. for pages A-1 and A-2, this number is 1X + 2Y+). The BODE number before the Y refers to the "ID" column in the table of calculated damping values on pages A-3 and A-4. This allows the correct correlation with appropriate table and plot. Appendix F provides a complete listing of all damping power spectras submitted by SDRC.

Figure 6 (a,b,c) shows a plot of amplitude versus frequency for the CORT, BLOUGH, and RYERSON respectively, which illustrates how the max amplitude under this procedure can be used to choose the appropriate damping factor. Notice that when all data is plotted as amplitude versus frequency, two peaks appear. This may be misleading since the peak of interest occurs at the known natural frequency for each vessel.

As mentioned before, the damping values calculated by SDRC represents the total system damping which includes hydrodynamic, structural and cargo damping. The representative system damping factor and frequency for the specific interval was chosen by picking the damping factor and frequency that corresponded to the max amplitude of the associated power spectrum. For example: the damping factor of 0.01247 chosen from page A-1 corresponds to a frequency of .3455 Hz. which corresponds to the max amplitude of 0.4804E-03 shown in the table of estimated roots (pg. A-3) and plotted on page A-2 (As mentioned in the cover letter from SDRC (Appendix B), the amplitude is in error by a constant factor).

In order to portray the complete picture at a glance, Tables 2, 3 and 4 were expanded to include other pertinent data such as ship course and speed, wind direction and speed, wave direction and height, max P-T stress, and etc. This data, extracted from References 1,2, and 3, is listed in Table 2, 3, and 4 to correspond to the CORT, BLOUGH, and RYERSON, respectively. Parentheses used in the Tables indicate that the data was extracted from the last recorded entry in that column from the original tables in References 1, 2, and 3.

Table #2 M/V STEWART J. CORT

Column	Tape #	Int.	Trip Date	Ship Course	Speed (MPH)	Prop. RPM	Sea State	Wind Direc-	Wind Speed -knots	Wave Direc-	Wave Ht. (ft)	Per. (sec)	Wave Length (ft)	Stresses (PSI) Max P-T	RMS	Max	ζ	w Hz.	Comments
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
High Springing -Load	18	20	12-13-73	009	-	084.1	-	002-R	29-R	-	-	-	-	14666	4560	17657	.0180	.20	----
	21	33	11-06-72	-	-	-	-	-	-	-	-	-	-	-	-	-	.0125	.33	----
	20	28	10-23-72	187	14	114.0	5	067-R	16-R	038 S	6	10	24	12236	4316	8960	.0246	.43	----
High Springing -Ballast	15	02	11-08-73	192	14	102	5	000-R	50-R	000	10	2	48	20306	4607	10037	.0125	.34	I
	15	02E	11-08-73	192	14	102	5	000-R	50-R	000	10	2	48	16464	5319	10850	.0181	.34	I
	19	24	11-20-74	302	14.4	-	5	005-T	34-T	070 S	8	-	-	20036	7299	-	.0146	.34	II
Medium Springing -Load	20	03	11-23-74	-	-	-	-	-	-	-	-	-	-	9150	3558	-	.00804	.28	----
	20	01	11-23-74	196	14.4	-	4	196-T	22-T	010 S	6	-	-	10167	4266	-	-	-	III
	19	05	11-17-74	205	14.1	-	4	210-T	30-T	000	6	-	-	9554	5197	-	.00828	.28	IV
Medium Springing -Ballast	15	03D	11-08-73	-	-	-	-	-	-	-	-	-	-	10896	4072	6122	.0167	.35	----
	15	06	11-08-73	-	-	-	-	-	-	-	-	-	-	9197	3647	3462	-	-	----
	17	27	12-04-73	266	15.2	101	3	090-R	12-R	135 S	4	2	15	9817	3727	4862	.00611	.32	V
Low Springing -Load	17	31	12-05-73	120	13	100	6	055-R	46-R	055 P	9	5	35	3985	1771	2684	.0137	.32	VI
	6	12	7-11-74	177	14.8	-	4	090-T	20-T	030 P	3	-	-	3935	1583	-	.0173	.28	VII
	8	21	8-11-74	114	13.3	-	6	130-T	28-T	015 S	4	-	-	4335	1730	-	.00719	.28	VIII
Low Springing -Ballast	9	10	8-21-74	292	14.6	-	3	218-T	14-T	060 P	3	-	-	5179	2094	-	.0122	.31	IX
	10	7	8-27-74	-	-	-	-	-	-	-	-	-	-	4382	2244	-	.0203	.31	----
	19	25	11-21-74	270	14.2	-	4	090-T	26-T	090 S	7	-	-	4595	1786	-	.0151	.30	X

Comments:
I - 29 mi. S. of Caribou Island
II - Manitou Island; 16 mi; 093
III - N. Manitou Island; 24 mi.; 334
IV - Pt. Betsie; 22 mi.; 290
V - 16 mi. E. of Taconite
VI - 8 mi. SE of Pic. Island, Lake Superior
VII - Burns Harbor; 32 mi.; 180
VIII - Crisp Pt.; 10 mi.; 171
IX - Crisp Pt.; 9 mi.; 220
X - Eagle Harbor; 25 mi.; 310

Table #3 M/V ROGER BLOUGH

Column	Tape # 1	Int. 2	Trip Date 3	Ship Course 4	Speed (MPH) 5	Prop. RPM 6	Sea State 7	Wind Direc- 8	Wind Speed -knots 9	Wave Direc- 10	Wave Ht. (ft) 11	Per. (sec) 12	Wave Length (ft) 13	Stresses (PSI) Max P-T 14	Stresses (PSI) RMS 15	Stresses (PSI) Max 16	ζ 17	ω Hz. 18	Comments 19
High Springing -Load	13	51	9-30-74	(078)	(15.8)	-	-	(350-T)	24-T	080 P	(4)	-	-	9074	3722	5369	-	-	I
	13	50	9-30-74	-	-	-	-	-	-	-	-	-	-	10830	3704	5879	.0171	.383	I
High Springing -Ballast	08R	36	8-31-74	(291)	(15)	-	-	(280-T)	(38-T)	(011 P)	(6)	-	-	20026	5651	8537	.0174	.42	II
	13R	45	10-17-74	(330)	(15)	-	-	(340-T)	(40-T)	(010 P)	(8)	-	-	16274	6372	7831	.0232	.39	III
	08R	35	8-31-74	291	15	-	-	280-T	38-T	011 P	6	-	-	14226	4948	7344	.033	.39	IV
Medium Springing -Load	13	52	9-30-74	(078)	(15.8)	-	-	(350-T)	(24-T)	(090 P)	(4)	-	-	7581	3422	4796	.0303	.382	I
	13	53	9-30-74	(078)	(15.8)	-	-	(350-T)	(24-T)	(090 P)	(4)	-	-	5961	2002	3531	.0248	.379	I
	13	47	9-30-74	(078)	(15.8)	-	-	(350-T)	(24-T)	(090 P)	(4)	-	-	7090	3067	3640	.0128	.387	I
Medium Springing -Ballast	13	03	9-29-74	(291)	(15)	-	-	(360-T)	(32-T)	(075 S)	(5)	-	-	10721	4568	6580	.0143	.41	V
	13	05	9-29-74	(291)	(15)	-	-	(360-T)	(32-T)	(075 S)	(5)	-	-	10603	3558	6352	.0227	.40	V
	13	04	9-29-74	(291)	(15)	-	-	(360-T)	(32-T)	(075 S)	(5)	-	-	10111	4223	6252	.0120	.41	V
Low Springing -Load	13	46	9-30-74	(078)	(15.8)	-	-	(350-T)	(24-T)	(090 P)	(4)	-	-	4459	1738	1893	.0202	.38	I
	13	43	9-30-74	(070)	(15.8)	-	-	(316-T)	(18-T)	(090 P)	(3)	-	-	4186	1665	2093	-	-	VI
	13	45	9-30-74	(078)	(15.8)	-	-	(350-T)	(24-T)	(090 P)	(4)	-	-	3777	1565	2229	.0103	.383	I
Low Springing -Ballast	08R	28	8-30-74	013	17	-	-	013-T	18-T	-	-	-	-	4273	1544	-	-	-	VII
	08R	38	8-21-74	291	15	-	-	299-T	20-T	010 P	6	-	-	4042	1581	1480	.0117	.43	VIII
	13R	57	10-17-74	(291)	(15.8)	-	-	(010-T)	(25-T)	(080 S)	(8)	-	-	4111	1823	3017	.0184	.42	IX

Comments:
I - (Eagle Harbor; 55 mi.; 270)
II - (Whitefish Point; 8 mi.; 315)
III - (Whitefish Point; 3 mi.; SE)
IV - Whitefish Point; 8 mi.; 315
V - (Whitefish Point; 10 mi.; 310)
VI - (Two Harbors; 15 mi.; 070)
VII - Gary Indiana; 21 mi.; 013; BP#2
VIII - Manitou Island; 35 mi.; 112; BP#3
IX - (Manitou Isalnd; 12 mi.; 090; BP#5

Table #4 S/S EDWARD L. RYERSON

Column	Tape # 1	Int. 2	Trip Date 3	Ship Course 4	Speed (MPH) 5	Prop. RPM 6	Sea State 7	Wind Direc- 8	Wind Speed -knots 9	Wave Direc- 10	Wave Ht. (ft) 11	Per. (sec) 12	Wave Length (ft) 13	Stresses (PSI) Max P-T 14	Stresses (PSI) RMS 15	Stresses (PSI) Max 16	ζ 17	w Hz. 18	Comments 19
High Springing -Load	5-1	1/1	11-22-67	-	17	105	7	058-R	30-R	058	3	-	10	-	-	11800	.00483	.57	I
	4-2	1/7	11-16-67	-	16.9	102	6	054-R	28-R	054	8	-	200	-	-	9110	.00455	.56	II
High Springing -Ballast	5-1	12/26	11-26-67	-	12	75	7	030-R	30-R	030	10	-	200	-	-	13420	.00433	.57	III
	5-1	13/27	11-26-67	-	12.2	75	7	030-R	30-R	030	12	-	200	-	-	12621	.00514	.57	IV
	5-1	15/28	11-26-67	-	11	75	7	014-R	34-R	025	12	-	200	-	-	13960	.00578	.57	V
	5-1	16/24	11-27-67	-	12	75	7	005-R	30-R	005	8	-	50	-	-	11800	.00522	.57	VI
Medium Springing -Load	5-2	19/1	11-27-67	-	17.1	106	5	133-R	26-R	133	4	-	75	-	-	7550	.00539	.56	VII
	5-2	24/6	11-28-67	-	17	105	7	032-R	35-R	032	5	-	50	-	-	7730	.00745	.56	VIII
	5-2	39/40	11-04-67	-	17.1	105	5	003-R	20-R	003	4	-	25	-	-	7090	.00395	.57	IX
Medium Springing -Ballast	4-1	10/18	11-12-67	-	16.7	98	6	037-R	26-R	015	6	-	100	-	-	6000	.00405	.58	X
	4-1	11/19	11-12-67	-	15	90	5	047-R	28-R	047	6	-	75	-	-	5900	.00362	.58	XI
	4-1	15/30	11-13-67	-	17	106	5	015-R	28-R	015	5	-	40	-	-	6540	.00387	.57	XII
	4-1	17/32	11-14-67	-	17.2	106	5	014-R	25-R	014	4	-	50	-	-	7940	.00475	.57	XIII
Low Springing -Load	6-1	8/15	12-09-67	-	17.1	105	4	097-R	20-R	097	4	-	30	-	-	4470	.00625	.57	XIV
	6-1	9/16	12-09-67	-	17.1	105	5	097-R	20-R	097	5	-	30	-	-	4390	.00495	.56	XV
	6-1	10/17	12-09-67	-	17	105	5	090-R	2o-R	080	5	-	75	-	-	4300	.00524	.44	XVI
Low Springing -Ballast	1-1	21/35	9-24-67	-	17.1	106	4	033-R	20-R	033	2	-	15	-	-	4170	.00359	.58	XVII
	1-1	24/40	9-25-67	-	17.5	106	4	075-R	24-R	075	4	-	50	-	-	3780	.00337	.59	XVIII

Comments:

I - Superior Shoals; 6 mi.
II - S. of Passage Island; 8 mi.
III - SE of Caribou Island; 21 mi.
IV - WSW of Caribou Island; 15 mi.
V - NNE of Manitou Island; 54 mi.
VI - SE of Passage Island; 9 mi.
VII - SE of Passage Island; 9 mi.
VIII - Straits of Mackinac
IX - N of Gull Island; 7.5 mi.
X - NE of Waukegan; 15 mi.
XI - NE of Milwaulkee; 15 mi.
XII - SE of Whitefish Point; 5 mi.
XIII - ENE of Manitou Island; 36 mi.
XIV - E of Port Washington; 30 mi.
XV - SE of Wind Point; 24 mi.
XVI - E of Glencoe, Illinois; 11 mi.
XVII - NE of Gull Island; 7 mi.
XVIII - SE of Caribou Island; 20 mi.

Method of Analysis

The system of analysis utilized by SDRC is known as MODAMS (MODal Analysis and Modeling System). The technique used to determine the desired modal parameter estimations was the multi-degree-of-freedom (MDOF) curve fit to the frequency response data over a specific frequency range. (see Appendix E for a brief description of a single d.o.f. system used by ABS)

The algorithm used by SDRC is explained in Appendix B of this report. This description gives the theoretical background for the curvefit algorithm used to extract modal parameters from the response spectra. This algorithm will work on spectra with no phase information but results in an analytical function of frequency whose calculated phase angle is essentially meaningless. Even though mode shape coefficients would also be meaningless, the mode shape evident in each graph is damping dependent and therefore allows the determination of the damping factor despite the lack of the phase information.

The basic concept behind the extraction of the damping values centers on the assumption that the operating data consists of a system's response to a fairly random forcing function (fairly flat frequency of input). The measured response can be strain, displacement, velocity, or acceleration. The first two are used directly and the latter two are converted to displacement data first. Since strain relates to displacement directly, the MDOF technique calculates $A\ell_k$ (page 7, Appendix B) from a displacement/force function. Since SDRC assumed the force applied was a constant function of frequency, using $F(w) = 1.0$ lbf for division in X/F did not affect the overall shape of the response curves.

Using this method allowed SDRC to establish the tables and associated graphs for each interval specified by the U. S. Coast Guard. The damping factor and frequency range chosen for each interval corresponds to the max amplitude for the frequency range investigated, specifically 0.01 to 1.0 Hz.

Analysis of SDRC Results

The range of values supplied by SDRC still raises the question of the accuracy of their results. Determining the accuracy may be difficult but determining whether or not they are in the "ballpark" would be highly beneficial.

The damping value (ζ) calculated by SDRC is the fraction of critical damping where:

$$\zeta = \frac{c}{c_c}$$

and: c = damping coefficient, tons-sec/in
c_c = critical damping coefficient, lb-sec/in
$= 2mw_n = 2\sqrt{mk}$
w_n = natural frequency = $\sqrt{k/m}$
k = stiffness factor, tons/in
m = mass, tons-sec^2/in

Then ζ represents a certain ratio of c to c_c to arrive at the damping coefficient c where $c_c * \zeta = c$.

In order to determine a "feel" for the approximate magnitude of the damping values for the three vessels, a mechanical method was used to determine the different springing parameters. This method utilizes the filtered records from existing stress records. It allows the determination of the approximate damping values from mechanical measurements taken directly from stress records.

Suppose that a free transient has the following form:

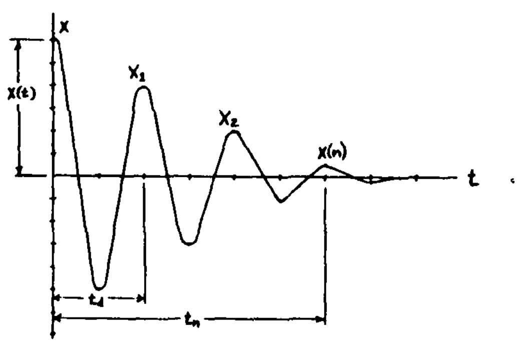

Figure 1 - Example of a Free Transient

Where: $X(t)$ = Amplitude at time t, psi
X_0 = Initial amplitude, psi, at t=0
X_1 = Amplitude after initial oscillation, psi
X_n = Amplitude after n oscillations
n = Number of oscillations
t_n = Time for n oscillations to occur, sec
t_d = Damped period, sec
w_d = Damped Frequency
w_n = Natural Frequency

The free transient response for this system shown in Figure 1 is:

$$X_0(t) = \frac{X_0 * e^{-\zeta\omega_n t} *(\sin(\sqrt{1-\zeta^2}\, w_n t + \phi))}{\sqrt{1-\zeta^2}}$$

With the initial conditions being:

$$X(0) = X_0$$

$$\dot{X}(0) = 0$$

These initial conditions are used in the mechanical analysis because the actual forces causing a peak condition is of no concern to this analysis. The area of interest is the decay period of the record. Once the peak springing stress is reached on a specific interval, the analysis is measuring the rate of decay as the oscillations decrease in amplitude.

From the springing record, X_0, X_n, and n may be measured. From these values the damping can be determined as follows:

$$\frac{X_0}{X_n} = \frac{X_0(0)}{X_n(t_n)} = \frac{X_0(0)}{X_0(n * t_d)} = e^{\frac{\zeta}{\sqrt{1-\zeta^2}} * 2\pi n}$$

and rearranging:

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{1}{2\pi n} * \ln \frac{X_0}{X_n}$$

When $\zeta << 1.0$, then $\sqrt{1-\zeta^2} \doteq 1.0$. Therefore:

$$\zeta = \frac{1}{2\pi n} * \ln \frac{X_0}{X_n} \qquad (1)$$

Equation 1 is the formula used to calculate the damping values found in Table 5. Appendix C contains the stress records from which the appropriate measurements were made in order to calculate damping via the mechanical method. From the same records the damped frequency (w_d), the natural frequency (w_n), the damped period (t_d), and the logarithmic decrement (δ) may be calculated via the following formulas:

$$w_d = \frac{n}{t_n} \text{ Hz.} = \frac{2\pi n}{t_n} \frac{\text{radians}}{\text{sec.}} \qquad (2)$$

$$w_n = \frac{1}{\sqrt{1 - \zeta^2}} * w_d \quad \text{Hz.} \tag{3}$$

Note: as per conditions for Equation 1, $w_d \approx w_n$

$$t_d = \frac{t_n}{n} \quad \text{sec.} \tag{4}$$

The term ln X_0/X_n from Equation 1 is often referred to as the logarithmic decrement and is defined as the natural logarithm of any two successive amplitudes. Since Table 5 lists the peaks amplitudes, X_0, and the amplitude after n oscillations, X_n, the equation for the logarithmic decrement is modified to be:

$$\delta = \frac{1}{n} * \ln \frac{Xo}{Xn} \tag{5}$$

These parameters are calculated from the measurements taken from Appendix C and the results are incorporated into Table 5. The areas of measurement are enclosed by double rectangles and are numbered to correspond to the numerical order in Table 5.

Logarithmic decrement may also be determined if the damping value is already known. The equation for δ can be written in the simplified form:

$$\delta = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}} = 2\pi\zeta \tag{6}$$

Equation 6 is an alternate method for determining δ when ζ is known and the stress records are not available for mechanical measurements as performed for Table 5.

Comparison of Results

In order to provide a means of comparison between the SDRC results and the mechanical results, a program was written in BASIC to calculate the δ , t_d, the bending moment (BM) from the data presented in Tables 2,3, and 4. A flow chart, the program data and the results are shown in Appendix D.

Table 5 - Damping Parameters Calculated from Mechanical Measurements

		X_0 psi	X_n psi	n cy	t_n (sec)	1 ζ	2 w_d Hz.	3 w_n Hz.	4 t_d sec/cy	5 δ	Ref. *
CORT	1	6206	2500	7	20.2	.0207	.35	.35	2.89	.129	C1:R9:14
	2	2327	517	5	13.4	.048	.37	.37	2.68	.30	C1 "
	3	4913	2586	6	17	.017	.35	.35	2.83	.107	C2 "
	4	6465	1552	5	14.6	.0434	.34	.34	2.9	.285	C2 "
	5	7758	1034	8	24	.040	.33	.33	3.0	.251	C3 "
	6	7500	3879	4	12.2	.026	.33	.33	3.05	.163	C3 "
	7	7241	1243	9	26.6	.0311	.34	.34	2.96	.191	C3 "
	8	6350	2533	5	17.6	.0293	.28	.28	3.52	.184	C4 "
	9	11200	1867	18	52.7	.0158	.34	.34	2.93	.099	C5:Q-L
	10	1917	800	6	17.6	.0232	.34	.34	2.93	.146	C6:Q-L
BLOUGH	11	6600	1000	12	30.0	.025	.4	.4	2.5	.025	C7:R2
	12	9250	2434	7	15	.0304	.47	.47	2.14	.191	C7 "
	13	5780	2720	6	16	.02	.38	.38	2.67	.126	C7 "
RYERSON	14	(.28)	-----	7	12.5	.0289	.56	.56	1.79	.182	C8:R3
	15	(.47)	----	5	9.0	.0240	.56	.56	1.80	.151	C8 "

*Reference Code C - Appendix C of this report
C1 - pg. 1 of Appendix C of this report
R9 - Reference 9 (page 19)
14 - pg. 14 of Reference
Q-L - Quick-look

**Numbers in parenthesis is % with X_0 being unit 1 and X_n being a fraction of X_0.

The locator code in the computer output refers to Tables 2,3, and 4. For example: the first locator code in the computer output (page D-4) is "CHSL*18*20". This refers to Table 2 meaning "CORT-High Springing Load, Tape #18, Interval #20". In this way a direct connection may be made between Tables 2,3, and 4 and the computer output. The equations used for the calculations of the variables in the program are located on page D-2.

Using the output from the computer, Tables 1 - 5, and the SDRC results, a comparison was performed to see how close all inputs agree. Figure 2 is a plot of δ versus ζ. The "Key" differentiates between the SDRC results and the mechanical results. Since ζ is small (i.e. $\ll$ 1.0), it was expected that the data points would lie on a straight line since δ is directly proportional to ζ as per Equation 6. The numbered points on Figure 2 correspond to the sequential numbering from Table 5. Generally the mechanical results plotted linearly with the SDRC results but gave higher values of ζ. This could possibly be explained by the lack of more information on the actual sea state existing at the time of stress measurements. Table 6 below gives the relative comparisons between the two results.

Table 6 - Comparison of SDRC Results with Mechanical Results

	SDRC		Mechanical		
	Range of ζ	Avg.	Range of ζ	Avg.	$\frac{\text{SDRC avg}}{\text{Mech. avg.}} * 100$
CORT	.00611-.0246	.014	.017-.048	.029	48 %
BLOUGH	.0103-.0248	.01916	.020-.0304	.0251	76 %
RYERSON	.00337-.00745	.00480	.024-.0289	.0265	18 %

As can be seen from Table 6, the differences in the calculated damping factors are quite pronounced, especially for the CORT and the RYERSON. At first inspection, this would tend to invalidate the SDRC results. However, despite the observed differences in Table 6 and Figure 2, an examination of frequency versus damping yields an interesting results.

Figure 3 is plot of w versus ζ using the results from SDRC. The data points for each vessel are enclosed within a dashed line. From Table 1 it may be seen that the CORT has a higher L/D ratio (22.2) than the other two vessels and this would tend to make the CORT more "flexible" than the BLOUGH or the RYERSON. This in turn would cause the CORT to have a lower springing frequency than the BLOUGH or the RYERSON. Since the RYERSON has the lower L/D ratio of 18.7, it would tend to be a stiffer ship and show a higher springing frequency. These observations are shown in Figure 3. The approximate springing frequency for the CORT is 0.32 Hz. despite the loading condition and despite the stress levels seen. By reviewing the original stress levels, the Low Springing Ballast case for the CORT is approximately one-third the levels in the High Springing Load case. (See Table 2) This is an important observation illustrating that exciters that would be used for establishing the fundamental mode of hull vibration (springing) would not have to generate peak stress levels. The box girder of the vessel would still spring at approximately 0.32 Hz. The damping would be measured more accurately since the out-of-phase seaway damping would essentially be eliminated.

Figure 4 is a plot of damping versus frequency as calculated in Table 5. The numbered points correspond to the sequential numbering in Table 5. The enclosed dashed lines represent the SDRC results in Figure 3. As shown, several points are outside the representative areas but the corresponding points do correspond to the same frequency as depicted in Figure 3. Comparison between Figure 3 and Figure 4 illustrates this notable point: using only analog stress records (digitized by Teledyne) during periods of vessel springing, SDRC developed a list of damping factors and associated frequencies at springing which matched "fairly" well with a mechanical analysis from apparently disassociated springing records (Appendix C) and agrees with the known natural frequency of each vessel.

The unknowns in both analyses are (1) the components of the total damping, (2) the method and distribution of seaway loading on the vessel, (3) the out-of-phase/in-phase characteristics of the seaway with the hull response, (4) understanding of the full complexity of the hull structure, and (5) more accurate and complete description of the sea conditions (i.e. wave height, wave length, wave direction, wind parameters, and actual loading conditions). With this as part of the total consideration, the results from SDRC are very encouraging despite the differences calculated in Table 6.

As one final attempt for correlation between the two methods, the program in Appendix D also calculated the bending moment, BM, using both the P-T stresses of each vessel from Appendix A, but also using the max stress, X_o, from Table 5. The calculated BM was plotted versus the damping factor. The plot is shown as Figure 5.

The values of BM calculated by Appendix D are shown in Column 6 of page D-4 of Appendix D. The values for BM from Table 5 are shown in Table 7. The BM was calculated by:

$$BM = \frac{SM * \sigma}{12 * 2000} \quad \text{FT-Tons}$$

where:

BM = total bending moment, ft-tons
SM = section modulus, in.^{3}
σ = stress, psi
12 = 12 in./ft.
2000 = 2000 lb./ton

Table 7 - Calculated Bending Moments from Initial Stress of Table 5

	X_o		BM	
1.	6206	PSI	23833	FT-TONS
2.	2327	"	8986	"
3.	4913	"	18868	"
4.	6465	"	24828	"
5.	7758	"	29794	"
6.	7500	"	28800	"
7.	7241	"	27808	"
8.	6350	"	24386	"
9.	11200	"	43012	"
10.	1917	"	7361	"
11.	6600	"	16500	"
12.	9250	"	23125	"
13.	5780	"	14450	"

The SM used for each vessel is shown on page D-2. Since the RYERSON worked by relative values of X_o and X_n, no BM was calculated. The calculated values for each vessel as per the SDRC results were boxed in. The BM's calculated in Table 7 were overlayed with the numbered points corresponding to the sequential numbering in Table 5 and in Table 7. There appears to be "general" agreement but there appears to be no conclusions that can be derived from this plot. The plotted points would be in better agreement if the damping factors were in better agreement. Therefore, the points for the BLOUGH show good agreement since the SDRC results are 76% of the mechanical results. (see Table 6) One final comment on Table 6: just because the results show better correlation of the SDRC results with the mechanical results for the M/V BLOUGH does not mean that a better analysis was performed on the BLOUGH or that better records were achieved or that springing was better identified. The only conclusion that can be drawn as result of Table 6 is that the comparative springing records in Appendix C may have more closely resembled the BLOUGH records used by SDRC. The sea conditions may have been closer to being the same. There is nothing to say that if different springing records had been chosen for comparison, the CORT or RYERSON results may have proven to give the best comparison with the SDRC results.

Conclusion

Considering the information that SDRC had and did not have, the results of their analysis is very encouraging. The damping values provided are a combination of structural damping, hydrodynamic damping, cargo damping, and out-of phase seaway damping. The data provided by SDRC may be used for further analysis. In order to arrive at the true structural damping within a vessel, an exciter capable of producing springing may be needed in order to eliminate so many of the unknowns which have complicated this analysis. The sign and operation of such a device presents its own difficulties in the form of unknowns, assumptions, and feasibility.

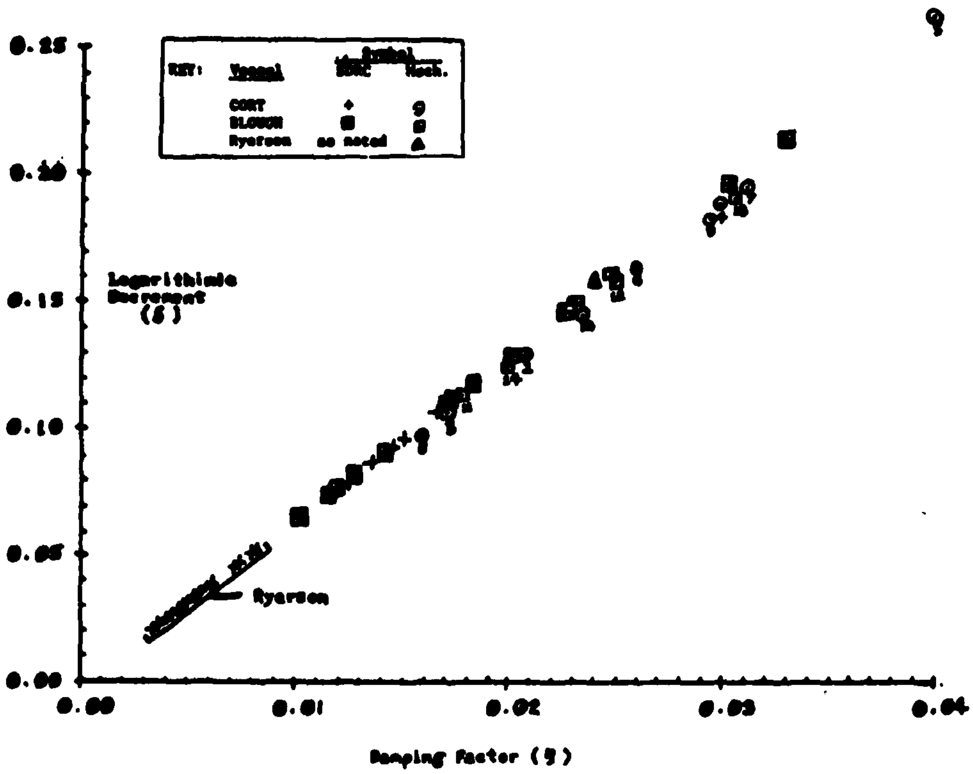

Figure 2 -Damping vs. Logarithimic Decrement - Comparison of SDRC and Mechanical Results

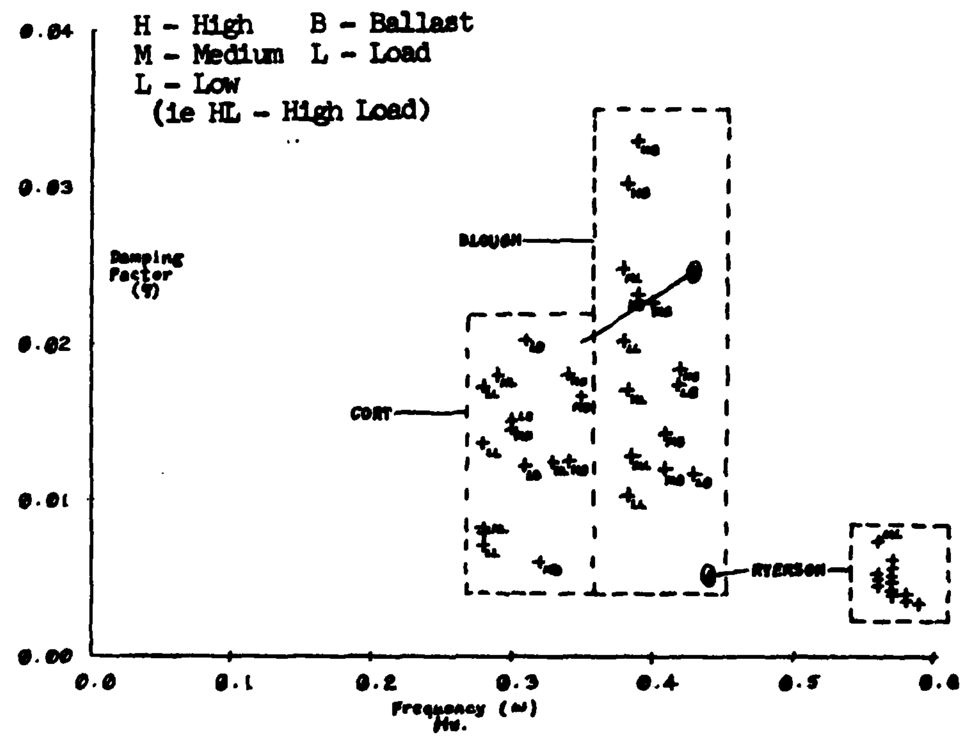

Figure 3 - Frequency vs. Damping Factor - SDRC data utilized in Calculations

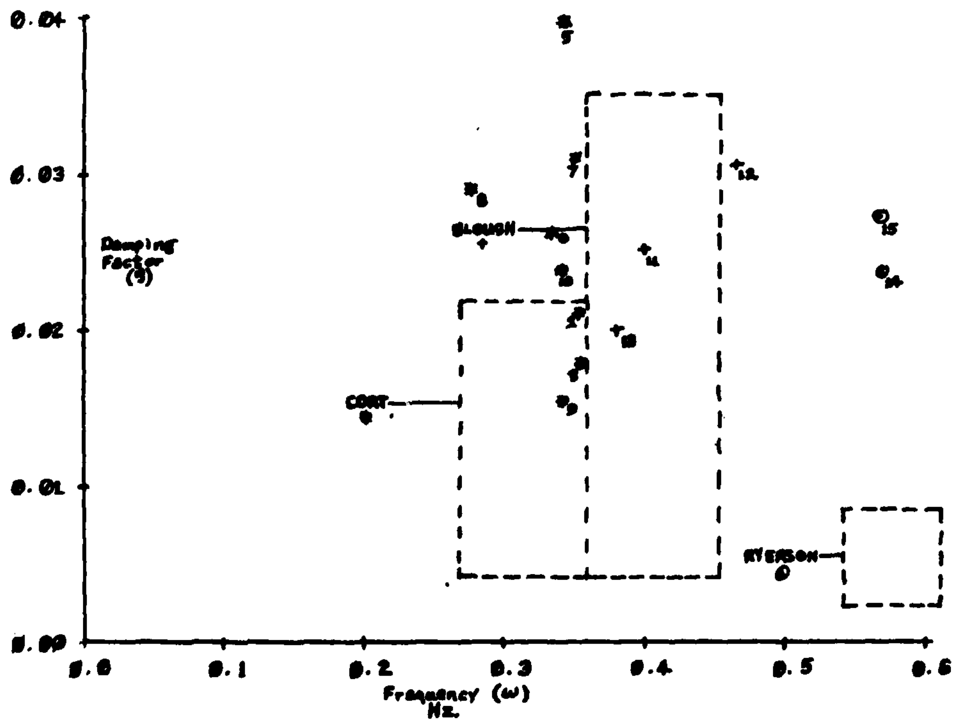

Figure 4 - Frequency vs. Damping Factor - Mechanical Data Used and Compared Figure 3

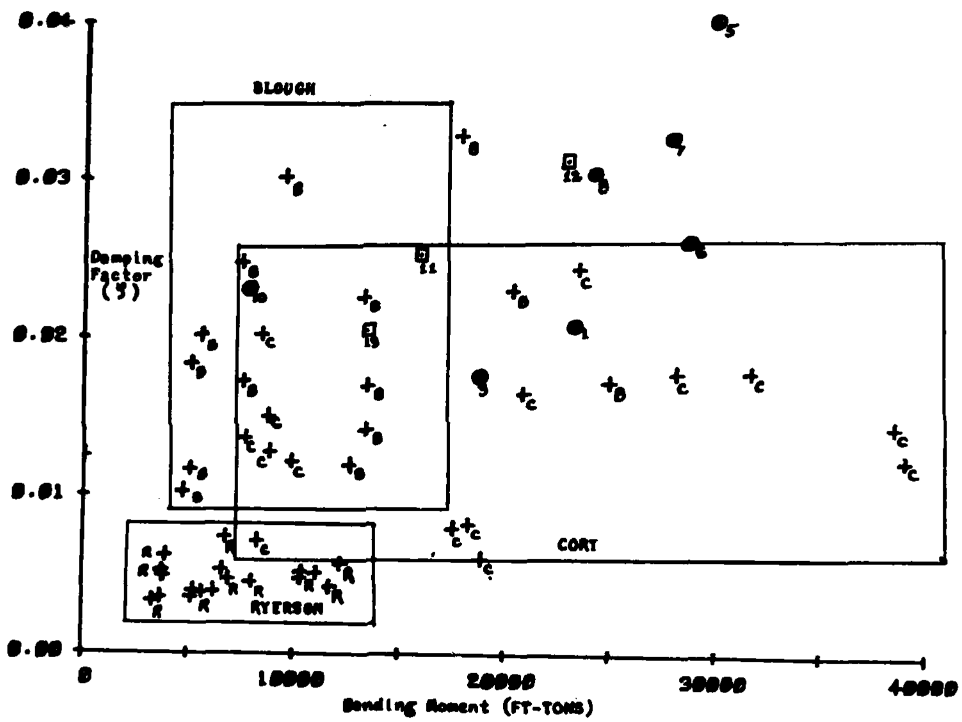

Figure 5 - Bending Moment vs. Damping Factor - Comparison of SDRC and Mechanical Results

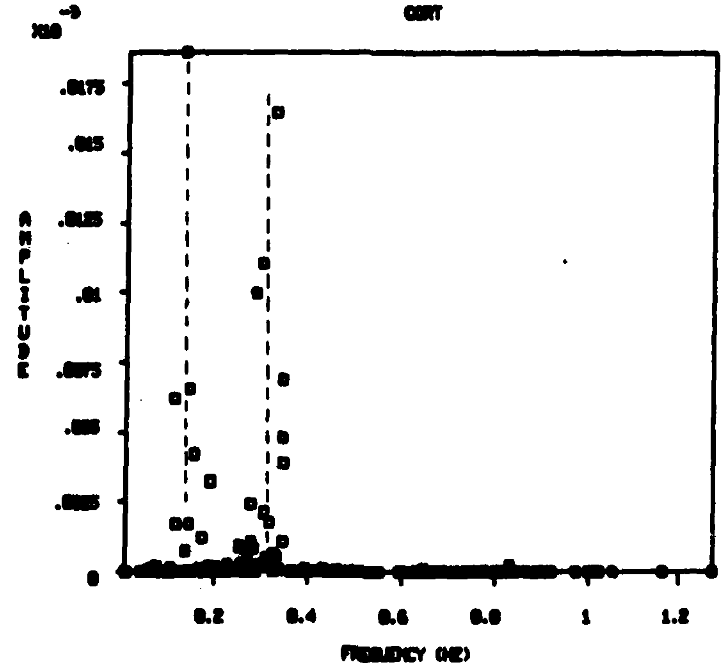

Figure 6(a) - Frequency vs. Amplitude - CORT
All Data Points Submitted by
SDRC

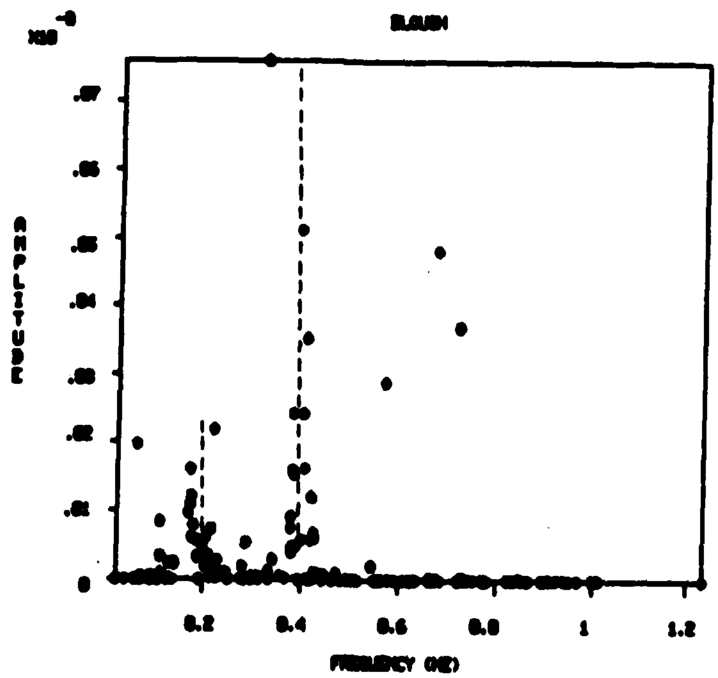

Figure 6(b) - Frequency vs. Amplitude - BLOUGH

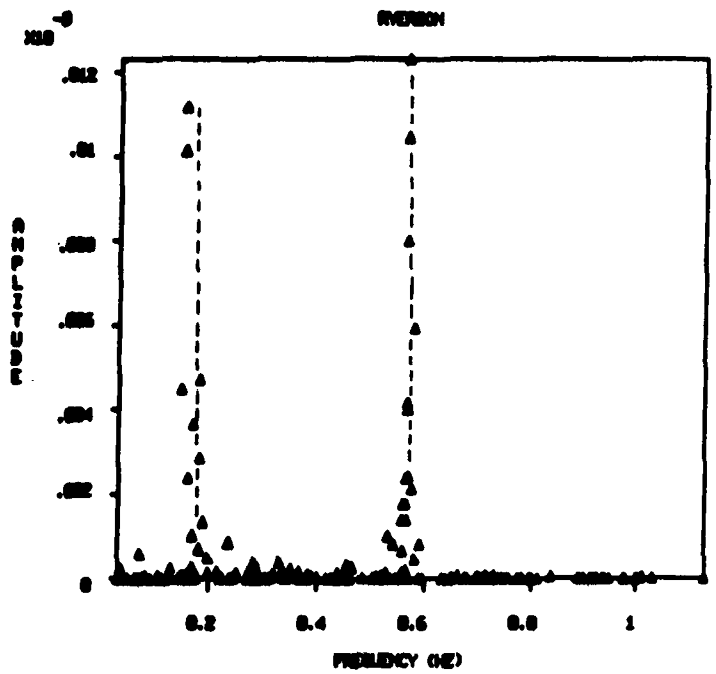

Figure 6(c) - Frequency vs. Amplitude - RYERSON

References

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2. "Instrumentation of M/V ROGER BLOUGH-Second Season (1974-1975)", June 6, 1975, Teledyne Materials Research, NTIS.#

3. "Measurement of Seaway Stresses Aboard Great Lake Ore Carrier EDWARDS L[RYERSON - (1967 Operating Season Only)", James W. Wheaton, Teledyne Materials Research, July 1968, SNAME Project E-1125(C).

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5. "Theory of Vibration with Applications", William T. Thompson, 1972.

6. "Damping Factors in the Higher Modes of Ship Vibration", Toyoji Kumai, Reports of Research Institute for Applied Mechanics, Vol. VI, No. 21, 1958.

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8. "Instrumentation for Mechanical Analysis", An intensive Short course notes, July, 12-16, 1976, Prof. Francis E. Fisher, Dept. of M. E. University of Michigan.

9. "Analysis of Springing Stresses on SS CHARLES M. BEEGHLY and M/V STEWART J. CORT" Final Report # CG-D-163-75, July 1975, NSRDC, NTIS AD-A-018589.

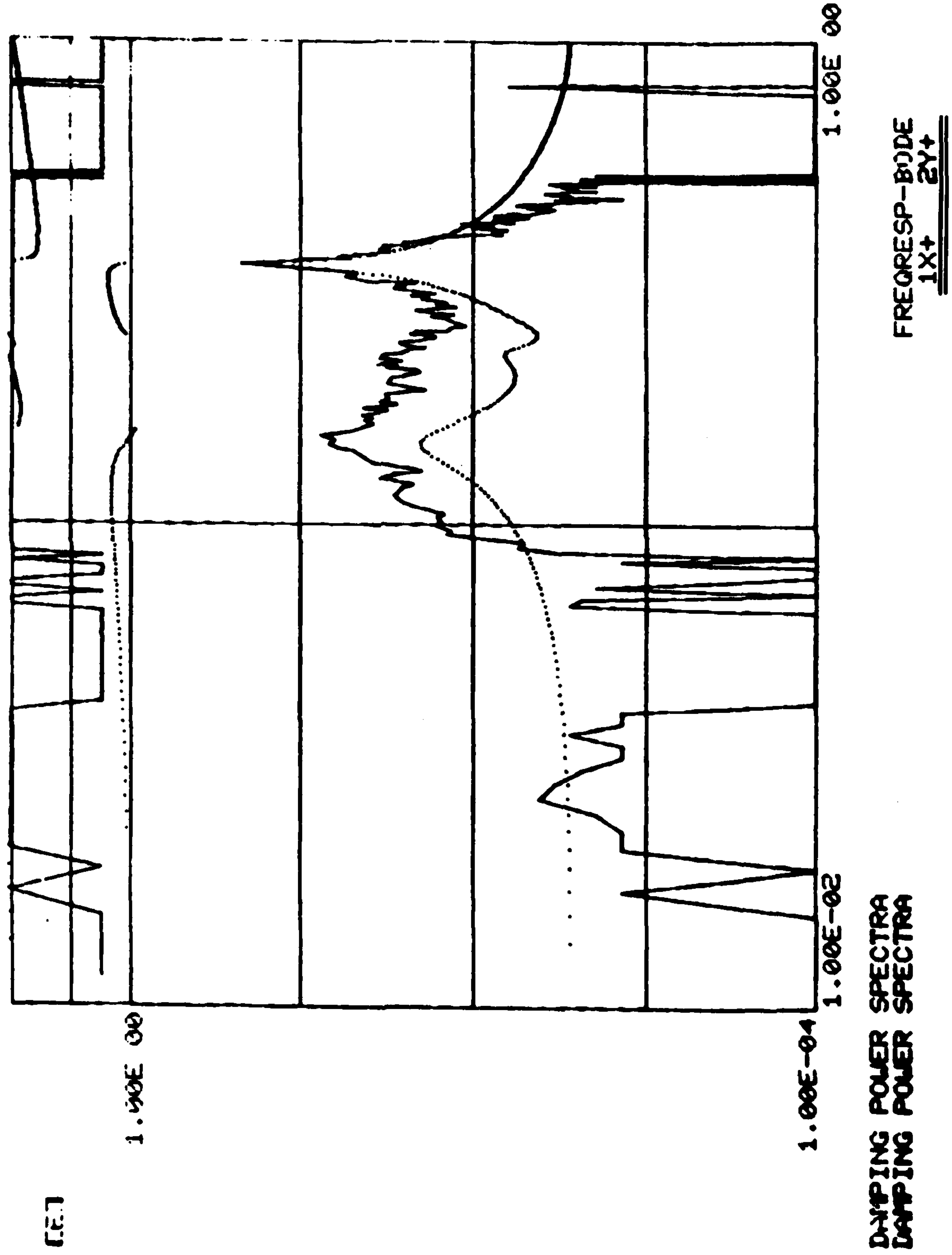
10. GenRad Brochure, "Modal Analysis... for improved structural design", obtained from Structural Dynamic Research Corp., Cincinnati, Ohio.

APPENDIX A

Sample Results from SDRC

ESTIMATED ROOTS (1X+ 2Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.5261E-02	1.000	0.9041E-06	-0.8724E-02
2	0.2469	1.000	0.1975E-08	2.902
3	0.1155	0.1981	0.1760E-05	-1.050
4	0.1476	0.8880E-01	0.1677E-03	0.2385
5	0.2305	0.5714E-01	0.2644E-04	-0.6653
6	0.3455	0.1247E-01	0.4804E-03	-0.2004E-01
7	0.3948	0.2279E-01	0.5835E-05	-1.366
8	0.4562	0.5420E-01	0.1298E-05	-1.783
9	0.5068	0.3782E-01	0.4574E-06	-0.9194
10	0.7963	0.3452E-01	0.2013E-09	1.323
11	0.8109	0.4167E-03	0.1914E-06	-0.6590E-02


1.00E 00
1.00E-04
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 2Y+

	Tape No.	ID		SDRC Reference Record No.	Track No.	Frequency (Hz)	% Critical Damping	USCG Interval	Number of Averages In PSD	Tape Count (ft.)
CORT	1	1	2	0	11	—	—	1	400	160-390
	1	2	2	1	3	.34	.0125	2	100	188-238
	1	3	2	2	3	.34	.0181	2E	100	443-493
	1	4	2	3	3	.35	.0167	3D	100	695-755
	1	5	2	4	11	.28	.00804	3	100	512-640
	1	6	2	5	10	.28	.0083	5	100	640-700
	1	7	2	6	3	—	?	6	60	850-900
	1	8	2	7	9	.31	.0203	7	100	2360-2513
	1	9	2	8	8	.31	.0122	10	50	975-1044
	1	10	2	9	6	.28	.0173	12	50	632-689
	1	11	2	10	4	.29	.0180	20	400	1245-1585
	1	12	2	11	7	.28	.00719	21	50	1130-1185
	1	13	2	12	10	.30	.0147	24	400	2130-2498
	1	14	2	13	10	.30	.0151	25	50	2500-2567
	1	15	2	14	5	.32	.00611	27	50	1338-1388
	1	16	2	15	1			28	50	1820-1907
	1	17	2	16	12	.43	.0246	28	50	1350-1400
	1	18	2	17	5	.32	.0137	31	50	1566-1615
	1	19	2	18	2	.33	.0125	33	400	3177-3534
BLOUGH	1	20	2	19	12	.39	.033	35	50	1699-1748
	1	21	2	20	12	.42	.0174	36	50	1748-1798
	1	22	2	21	12	.43	.0117	38	50	1848-1898
	1	23	2	22	14	.39	.0232	45	50	2395-2444
	1	24	2	23	14	.42	.0184	57	50	3044-3093
RYERSON	2	25	2	24	4	.57	.00483	1	100	0-100
	2	26	2	25	5	.56	.00539	1	300	26-315
BLOUGH	2	27	2	26	7	.41	.0143	3	50	150-196
	2	28	2	27	7	.41	.0120	4	50	200-248
	2	29	2	28	7	.40	.0227	5	50	250-292
RYERSON	2	30	2	29	5	.56	.00745	6	200	515-650
	2	31	2	30	3	.56	.00455	7	100	570-630
	2	32	2	31	6	.57	.00625	15	50	1270-1310
	2	33	2	32	6	.56	.00495	16	50	1320-1360
	2	34	2	33	6	.44	.00524	17	50	1370-1410
	2	35	2	34	2	.58	.00405	18	100	1650-1705
	2	36	2	35	2	.58	.00362	19	300	1715-1840
	2	37	2	36	4	.57	.00433	26	400	1843-1995
	2	38	2	37	4	.57	.00514	27	400	2200-2375

RYERSON	2	39	2	38	4	.57	.00578	28	400	2570-2725
	2	40	2	39	4	.57	.00422	29	100	2930-2993
	2	41	2	40	2	.57	.00387	30	100	2535-2585
	2	42	2	41	2	.57	.00475	32	200	2670-2755
	2	43	2	42	1	.58	.00359	35	50	1905-1945
	2	44	2	43	5	—	—	39	200	3170-3255
	2	45	2	44	5	.57	.00395	40	200	3320-3410
	2	46	2	45	1	.59	.003365	40	50	2140-2200
BLOUGH	2	47	2	46	7	—	—	43	50	2240-2275
	2	48	2	47	7	.383	.0103	45	50	2338-2373
	2	49	2	48	7	.380	.0202	46	50	2395-2425
	2	50	2	49	7	.387	.0128	47	50	2440-2475
	2	51	2	50	7	.383	.0171	50	50	2590-2625
	2	52	2	51	7	—	—	51	50	2640-2680
	2	53	2	52	7	.382	.0303	52	50	2690-2730
	2	54	2	53	7	.379	.0248	53	50	2740-2780

APPENDIX B

Theoretical Background for Curve Fit Algorithm

SDRC
Structural Dynamics Research Corporation
5729 Dragon Way
Cincinnati, Ohio 45227
513-272-1100 TWX 810-461-2615

August 15, 1977

Lt. M. Noll
Commandant (G-DSA/TP44)
U.S. Coast Guard
Washington, D.C.

Subject: Results of Springing Damping Analysis, SDRC Proposal No. 6704-01; Your RFQ No. G-FCP-13/C22/77.

Dear Mark:

Attached you will find:

1. Table of calculated damping values.
2. Tabulation of modal parameters from each curve-fit
3. Plots with superposition of data and curve fit
4. Theoretical background for curve-fit algorithm

There are a few comments I would like to make concerning the interpretation of results:

1. The curve-fit algorithm used on this data to extract modal parameters is briefly described in the attachment (4). The form of the data we curve-fit was PSD, of course, not transfer functions. However, the data was assumed to be a transfer function where the force applied was a constant function of frequency, F(w) = 1.0 lbf so that division by this to get X/F (actually strain/force, which is not an important difference here) will not affect the shape of the curves.
2. The algorithm we use to curve fit transfer functions will work on spectra with no phase information. The result is simply an analytical expression as a function of frequency whose amplitude is in error by a constant factor, and of course the calculated phase angle is meaningless. Thus mode shape coefficients calculated from a power spectrum would be meaningless. The damping on the other hand does not suffer from the lack of phase angle. As the superimposed plots show the overall shape of the curves, which are damping dependent, are correct, but the plot must be shifted up or down the vertical axis to obtain the correct amplitude. The constant by which the curve is in error depends upon the frequency of the most dominant peaks, so not all the plots show the same amplitude error.
3. The Hilbert transform is used to calcualte a phase angle from a power spectrum as a function of frequency. The inclusion of the phase angle data with the amplitude data in the curve fit allows exact amplitude correlation between transfer functions calcualted from the curve fit parameters and the original data (see Figures 1 and 2). However, as the enclosed plots show, the visual correlation can be done very well by comparing the shapes of the two curves.

4. The reason the Hilbert transform was not utilized in this project is that hardware problems with the computer were inhibiting a function required in the phase angle computation. For expediency, I elected to proceed with the data reduction rather than await the correction problem. The algorithm for curve-fitting was checked out on other data prior to reduction of the Coast Guard data to verify the hardware problem was not interfering with its operation.

If you need more information, or have any questions, please contact me.

Regards,

R. Gene Smiley

R. Gene Smiley
Project Manager

RGS/dh

Enclosures

THEORETICAL BACKGROUND FOR CURVE-FIT ALGORITHM

APPENDIX A - THEORETICAL BACKGROUND

The theory behind modal analysis via frequency response functions can be examined by referring to the equations of motion for an N degree of freedom system with viscous damping:

$$[M][\ddot{q}] + [C][\dot{q}] + [K][q] = [f] \tag{1}$$

where

$[M]$ = mass matrix

$[C]$ = viscous damping matrix

$[K]$ = stiffness matrix

$[q]$ = time history of the displacement of system

$[f]$ = time history of excitation to system

$[\dot{q}]$ = time history of velocity of system

$[\ddot{q}]$ = time history of acceleration of system

This equation is inconvenient to handle with standard methods of eigenvalue analysis if $[C]$ is not proportional to $[M]$ or $[K]$. However, a method has been proposed by Duncan (1)* which reduces these equations to a standard eigenvalue form. In this method combine the identity:

$$[M][\dot{q}] - [M]\,[\dot{q}] = [0]$$

with Equation 1 to obtain:

$$\begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix} \begin{bmatrix} [\ddot{q}] \\ [\dot{q}] \end{bmatrix} + \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix} \begin{bmatrix} [\dot{q}] \\ [q] \end{bmatrix} = \begin{bmatrix} [0] \\ [f] \end{bmatrix} \tag{2}$$

Represent this equation in the following manner:

$$[A][\dot{y}] + [B][y] = [z] \tag{3}$$

*Numbers in parentheses designate references at end of section where

$$[A] = \begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix}$$

$$[B] = \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix}$$

$$[y] = \begin{bmatrix} [\dot{q}] \\ [q] \end{bmatrix}$$

$$[z] = \begin{bmatrix} [0] \\ [f] \end{bmatrix}$$

In order to find the solution to Equation 3 for the case of harmonic inputs, first consider the solution to the homogeneous equation found by letting $[z] = [0]$

$$[A][\dot{y}] + [B][y] = [0] \tag{4}$$

Seek a solution of the form $[y] = [Y]e^{st}$

therefore $[\dot{y}] = s\,[Y]e^{st}$

Hence, Equation 4, becomes,

$$s[A][y] + [B][y] = [0] \text{ or}$$

$$\Big[[B] + s[A]\Big]\ [y] = [0]$$

This set of equations only have a solution if the determinant of the coefficient matrix is zero.

$$\det \Big\{[B] + s[A]\Big\} = 0$$

This leads to a set of 2N roots or eigenvalues $s_1, s_2 \ldots . s_{2n}$, which satisfy the above equation. For a resonant system, these eigenvalues will occur in conjugate pairs (2). Corresponding to each eigenvalue s_r, there exists an eigenvector $[\Psi^r]$ having 2N components satisfying the following equation:

$$\Big[[B] + s_r[A]\Big]\ [\Psi^r] = [0] \tag{5}$$

In the case where the eigenvalues of a system are complex, in which case they occur in conjugate pairs, the eigenvectors will be complex and will also occur in conjugate pairs,

The above eigenvectors have important orthogonality conditions which can be easily shown. Consider the r^{th} and p^{th} eigenvectors $[\Psi^r]$ and $[\Psi^p]$ both of which satisfy Equation 5. First write Equation 5 for the r^{th} mode and premultiply by the transposed vector $[\Psi^p]^T$, to obtain:

$$[\Psi^p]^T[B][\Psi^r]-s_r[\Psi^p]^T[A][\Psi^r]=[0] \tag{6}$$

Using the reversal law for transposed matrix products and recalling that [A] and [B] are symmetric matrices, transpose Equation 6 to obtain:

$$[\Psi^r]^T[B][\Psi^p]+s_r[\Psi^r]^T[A][\Psi^p]=[0] \tag{7}$$

Next write Equation 5 for the p^{th} mode and premultiply by $[\Psi^r]^T$

$$[\Psi^r]^T[B][\Psi^p]+s_p[\Psi^r]^T[A][\Psi^p]=[0] \tag{8}$$

If Equation 8 is subtracted from Equation 7, the result is:

$$(s_r-s_p)\ [\Psi^r]^T[A][\Psi^p] = 0$$

If eigenvalues s_r and s_p are different, the following orthogonality property relates the two eigenvectors.

$$[\Psi^r]^T[A][\Psi^p] = 0 \tag{9}$$

It follows that these vectors are also orthogonal with respect to matrix [B]

$$[\Psi^r]^T[B][\Psi^p] = 0 \tag{10}$$

Equations 9 and 10 are important orthogonality conditions which shows that the 2N vectors $[\Psi]$ form a linearly independent set, and therefore any vector in 2N space can be expressed as a linear combination of these 2N vectors. Since we are interested in frequency response information, let

$$[z(t)]=[Z]\, e^{j\omega t} \tag{11}$$

and seek a solution in the form:

$$[y(t)] = [Y] e^{j\omega t} \tag{12}$$

Substitute Equation 11 and Equation 12 into Equation 3 and divide by $e^{j\omega t}$ to obtain:

$$j\omega[A][Y] + [B][Y] = [Z] \tag{13}$$

Since the eigenvectors defined by Eqution 5 form a linearly independent set over 2N space, write the solution to Equation (13) as a linear combination of these 2N vectors

$$[Y] = \sum_{r=1}^{2N} \gamma_r[\Psi^r] \tag{14}$$

Substitute Equation 14 into Equation 13 and multiply by $[\Psi^p]^T$ to obtain:

$$j\omega[\Psi^p]^T[A] \sum_{r=1}^{2N} \gamma_r[\Psi_r]+[\Psi^p]^T[B] \sum_{r=1}^{2N} \gamma_r[\Psi^r]=[\Psi^p]^T[Z]$$

From the orthogonality conditions, we obtain:

$$j\omega a_p(\gamma_p) + b_p(\gamma_p)=[\Psi^p]^T[Z] \tag{15}$$

where

$$a_p = [\Psi^p]^T[A][\Psi^p]$$
$$b_p = [\Psi^p]^T[B][\Psi^p]$$

Hence, we can solve Equation 15 for γ_p to obtain:

$$\gamma_p = \frac{[\Psi^p]^T[z]}{j\omega a_p + b_p} \qquad (16)$$

Substituting Equation 16 into Equation 14, we obtain:

$$[Y] = \sum_{r=1}^{2N} \frac{[\Psi^r]^T[z][\Psi^r]}{j\omega a_r + b_r} \qquad (17)$$

However, from Equation 7, we obtain

$$b_r + s_r a_r = 0$$

$$s_r = \frac{-b_r}{a_r}$$

Therefore, Equation (17) can be written

$$[Y] = \sum_{r=1}^{2N} \frac{[\Psi^r]^T[z][\Psi^r]}{a_r(j\omega - s_r)} \qquad (18)$$

Frequently the complex eigenvalues s_r are written in the following form:

$$s_r = -\zeta_r \omega_r \pm j \omega_r\sqrt{1-\zeta_r^2}$$

where

ζ_r = damping ratio

ω_r = undamped natural frequency

In terms of [Q] and [F], Equation 18 becomes:

$$\begin{bmatrix} j\omega Q \\ Q \end{bmatrix} = \sum_{r=1}^{2N} \frac{[\Psi^r]^T\begin{bmatrix} 0 \\ F \end{bmatrix}[\Psi^r]}{a_r(j\omega - \zeta_r\omega_r \pm j\omega_r\sqrt{1-\zeta_r^2})}$$

Therefore the frequency response function recorded from excitation applied at location k and response monitored at location i is:

$$H_{ik}=\sum_{r=1}^{2N}\frac{\Psi_k^r\ \Psi_i^r}{a_r(j\omega+\zeta_r\omega_r\pm j\omega_r\sqrt{1-\zeta_r^2})} \tag{19}$$

Since the eigenvalues occur in conjugate pairs, Equation 19 can be written as

$$H_{ik}=\sum_{r=1}^{N}\frac{\Psi_k^r\ \Psi_i^r}{a_r(j\omega+\zeta_r\omega_r+j\omega_r\sqrt{1-\zeta_r^2}}+$$

$$\frac{\Psi_k^{r*}\ \Psi_i^{r*}}{a_r^*(j\omega+\zeta_r\omega_r-j\omega_r\sqrt{1-\zeta_r^2})} \tag{20}$$

Equation 19 is an extremely valuable relationship between Frequency Response Functions and modal characteristics. It relates motion at any point i due to an force at point k. Notice that equation 19 implies that the frequency response between response at i and excitation at k is the same as the function between response at k and excitation at i. Equation 19 is frequently written in the form:

$$H_{ik}=\sum_{r=1}^{2N}\frac{A_{ik}^r}{(s-s_r)}=\sum_{r=1}^{N}\frac{A_{ik}^r}{(s-s_r)}+\frac{A_{ik}^{r*}}{(s-s_r^*)} \tag{21}$$

where

A_{ik}^r = Residue at pole s_r(i.e. $\frac{\Psi_i^r\ \Psi_k^r}{a_r}$)

The impulse response of the system can be obtained from Equation 21 by performing an inverse transform to obtain:

$$H_{ik}(t)=\sum_{r=1}^{2N}A_{ik}^r\ e^{s_r t} \tag{22}$$

Since the roots occur in conjugate pairs, Equation 22 can be written in the form:

$$H_{ik}(t) = 2 \sum_{r=1}^{N} \left| A^r_{ik} \right| e^{-\zeta_r \omega_r t} \cos\left[\left(\omega_r \sqrt{1-\zeta_r^2} \right) + \phi^r_{ik} \right] \quad (23)$$

where

$$\phi^r_{ik} = \measuredangle A^r_{ik}$$

Equation 23 indicates that the impulse response of the system can be represented by a summation of the number of damped cosine waves times the appropriate modal parameters.

The multi-degree-of-freedom (MDOF) curve fitting procedures in the modal analysis program calculates the value of A^r_{ik} in the above equations. Therefore, in the case where A^r_{ik} was determined from a displacement/force frequency response function, the value of a_r can be determined from the equation:

$$a_r = \frac{\Psi^r_i \quad \Psi^r_k}{A^r_{ik}}$$

where

A^r_{ik} is determined from a displacement/force function.

In the case where a velocity/force frequency response function was curve fit with the MDOF procedure, the parameter a_r is determined from the following:

$$a_r = \frac{\Psi^r_i \quad \Psi^r_k}{A^r_{ik}} \times j \; \omega_r$$

where: ω_r is in the units of rad/sec and

A^r_{ik} is determined from a velocity/force function

Similarly, if an acceleration/force frequency response function is used, a_r is determined from the equation:

$$a_r = \frac{\Psi_i^r \ \Psi_k^r}{A_{ik}^r} \times (-\omega_r^2)$$

where

ω_r is in units of rad/sec and

A_{ik}^r is determined from an acceleration/force function

If an analytical model is to be created from the test data, the parameters a_r, Ψ^r, ω_r and ζ_r are all that is necessary to describe the component with complex normal modes. However, in some cases an analyst would like to use a "real" mode approximation with the associated effective mass or effective stiffness in order to describe the component under test via the following equation:

$$H_{ik} = \sum_{r=1}^{N} \frac{\Psi_i^r \quad \Psi_k^r}{m_r[\omega_r^2 - \omega^2 + j\, 2\, \zeta_r \omega \omega_r]} \tag{24}$$

In that case it is recommended that this approximate representation be determined by setting the magnitude of the mode shape coefficient equal to the magnitude of the complex mode shape value and the sign of the mode shape coefficient from one of the following procedures:

1) Inverse of the sign of the imaginary portion of the mode shape coefficient when a displacement/force frequency

response function is used to determine the mode shape coefficients.

2) The sign of the real portion of the mode shape coefficient when a velocity/force frequency response function is used to define the mode shape coefficients.

3) Sign of the imaginary portion of the mode shape coefficient when an acceleration/force frequency response function is used to determine the mode shape coefficients.

The effective mass necessary to approximate the actual frequency response with that described by Equation 24 can be determined from one of the following equations:

$$m_r = \frac{(\text{Approx. } \Psi_i^r)(\text{Approx. } \Psi_k^r)}{2\omega_r \left| A_{ik}^r \right|}$$

where A_{ik}^r is determined from a displacement/force functic

$$m_r = \frac{(\text{Approx. } \Psi_i^r)(\text{Approx. } \Psi_k^r)}{2 \left| A_{ik}^r \right|}$$

where A_{ik}^r is determined from a velocity/force function

$$m_r = \frac{(\text{Approx. } \Psi_i^r)(\text{Approx. } \Psi_k^r)}{2 \left| A_{ik}^r \right|} \times \omega_r$$

where A_{ik}^r is determined from a acceleration/force functic

In order to represent a component in an overall system model via a "real" mode approximation, the following approach can frequently be used. The uncoupled equations of motion for the component in terms of modal coordinates [γ] are:

$$\left\{-\omega^2[\lceil m \rfloor] + j\omega[\lceil c \rfloor] + [\lceil k \rfloor]\right\}[\gamma] = [F_s]$$

where $[\lceil m \rfloor]$ is a diagonal matrix of effective masses

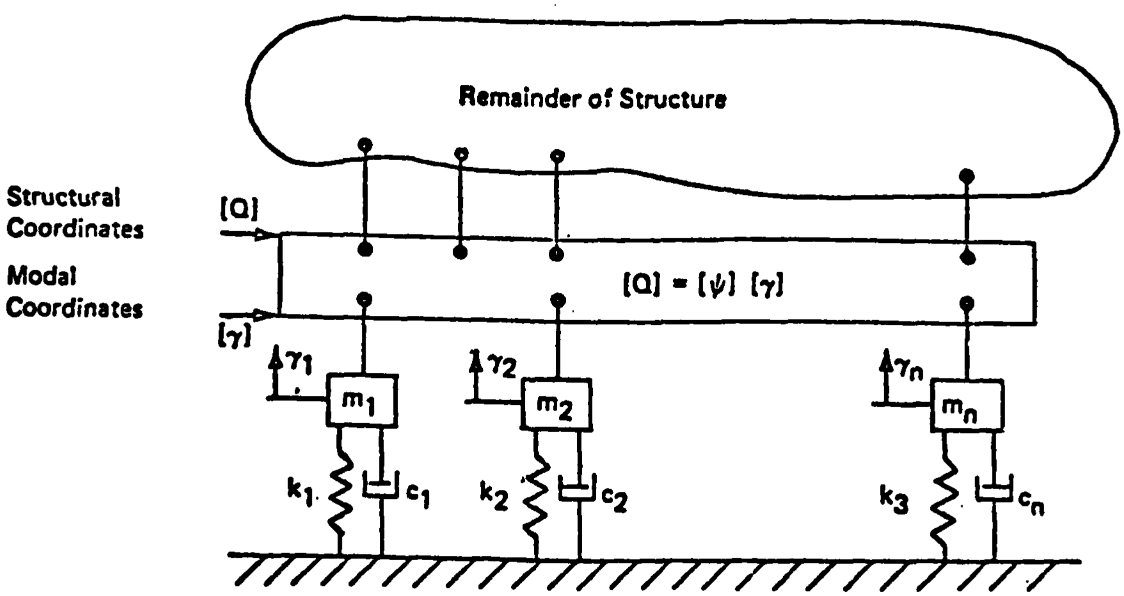
$[\lceil c \rfloor]$ is a diagonal matrix of effective dampness

$[\lceil k \rfloor]$ is a diagonal matrix of effective stiffnesses

The motion of the physical coordinates [Q] is related to the motion of the modal coordinates by:

$$[Q] = [\Psi]\,[\gamma]$$

Symbolically, this can be represented by the following diagram:



Therefore, a component can be represented analytically from test data in an overall system model by a set of springs, masses, dampers and equations of constraint which relate the motion of the physical coordinates to the motion of the modal coordinates. Since the equations of constraint can be quite voluminous, the NASTRAN input and MATRIX Generation task provides the capability to generate NASTRAN Multi Poin Constraint (MPC) equations in a relatively automatic manner.

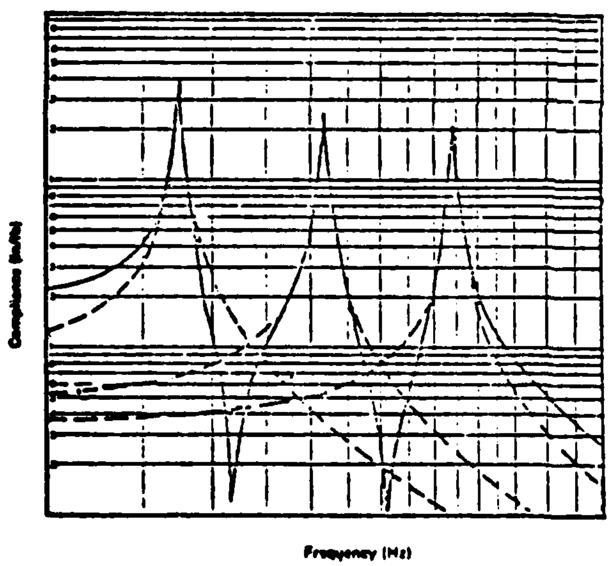
The frequency response in a specified range can be approximately described in terms of the following quantities:

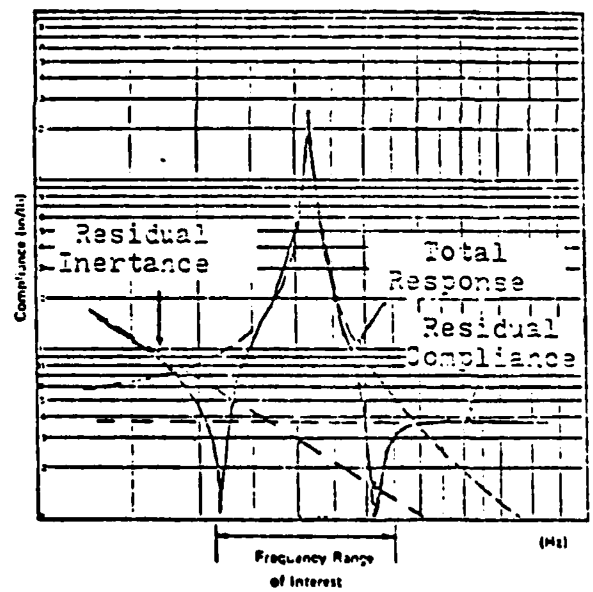
1) "Residual Inertance" of the modes of vibration below the range of interest.
2) The modes of vibration which are resonant in the specified frequency range.
3) "Residual Compliance" of the modes of vibration above the range of interest.

Mathematically this can be expressed as:

$$H_{ik} = \frac{X_{ik}}{\omega^2} + \sum_{r=1}^{2N} \frac{\Psi_k^r \Psi_i^r}{a_r(j\omega + \zeta_r\omega_r \pm j\omega_r\sqrt{1-\zeta_r^2})} + Z_{ik}$$

This concept is shown graphically in the following figures:





The SDOF and MDOF curve fitting procedures available in the Estimation task are used to evaluate the contribution due to the modes which are resonant in the frequency range under investigation. In order to determine the contribution of the residual effects the Generate Residual command in the Frequency Response Synthesis task is used.

REFERENCES

1. W. J. Duncan, "Elementary Matrices" Macmillian Company, New York 1946

2. W. C. Hurty, "Dynamics of Structures" Prentice Hall Inc. 1964

3. A. L. Klosterman, "On the Experimental Determination and Use of Modal Representations of Dynamic Characteristics", Ph.D. Dissertation, University of Cincinnati, 1971.

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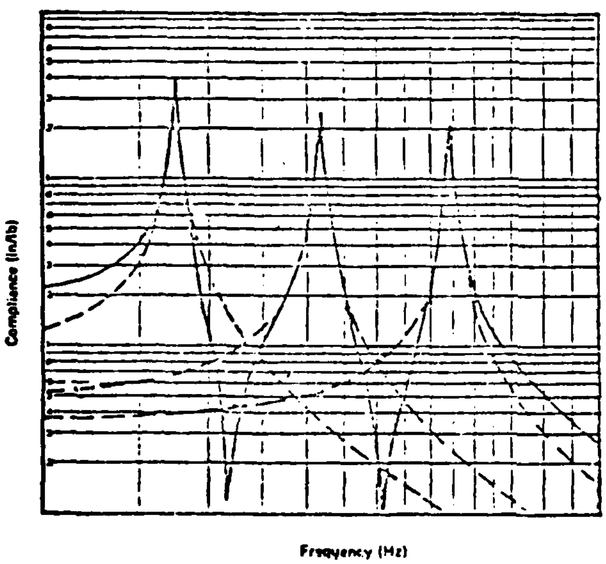

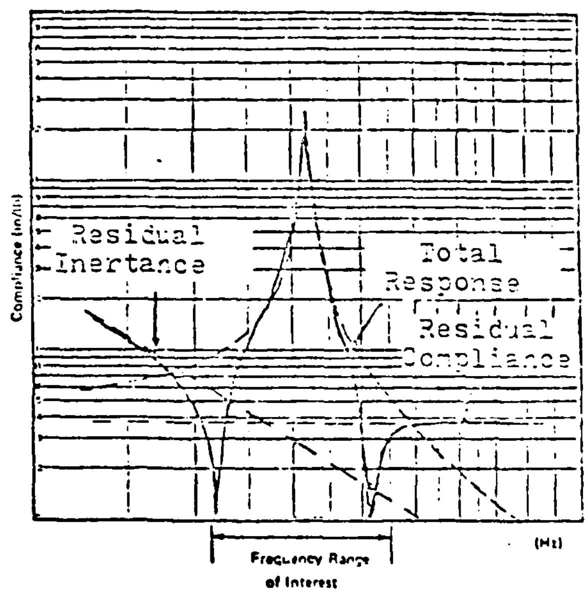

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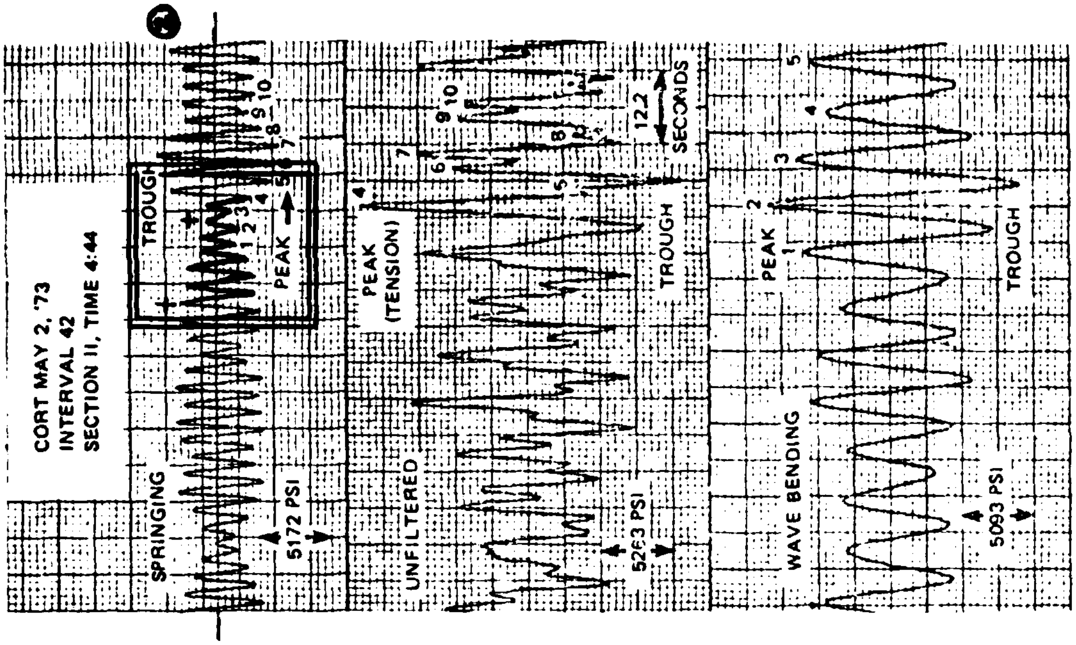
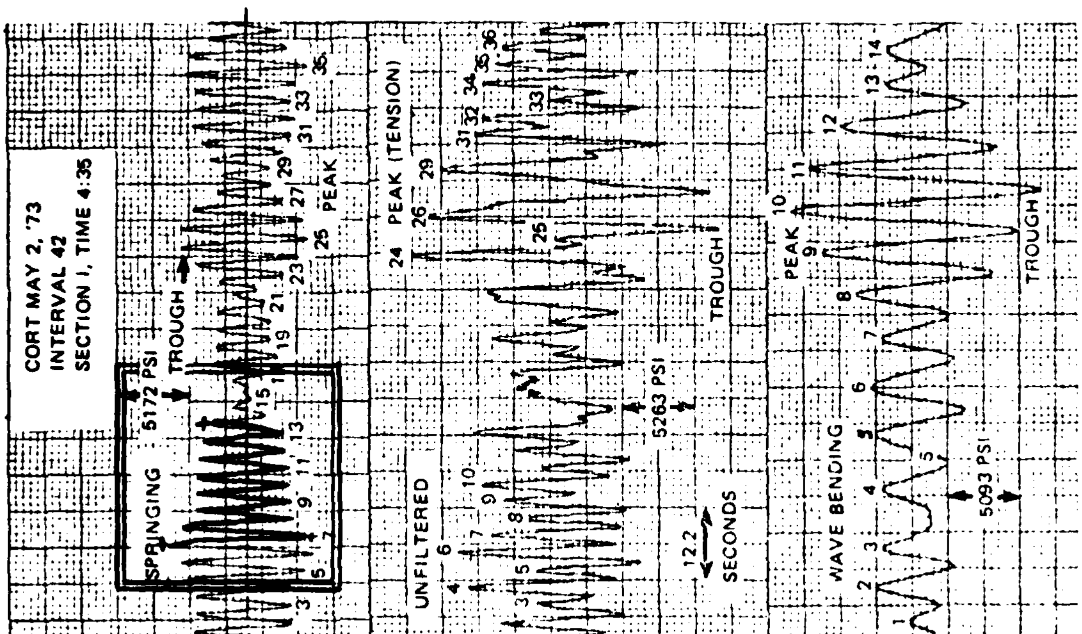
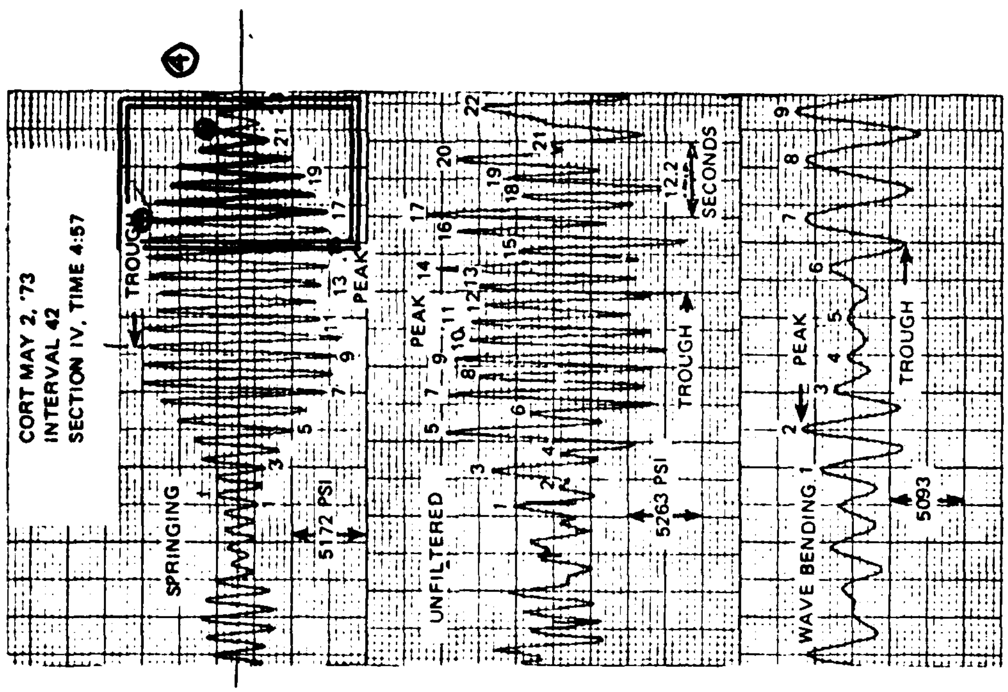
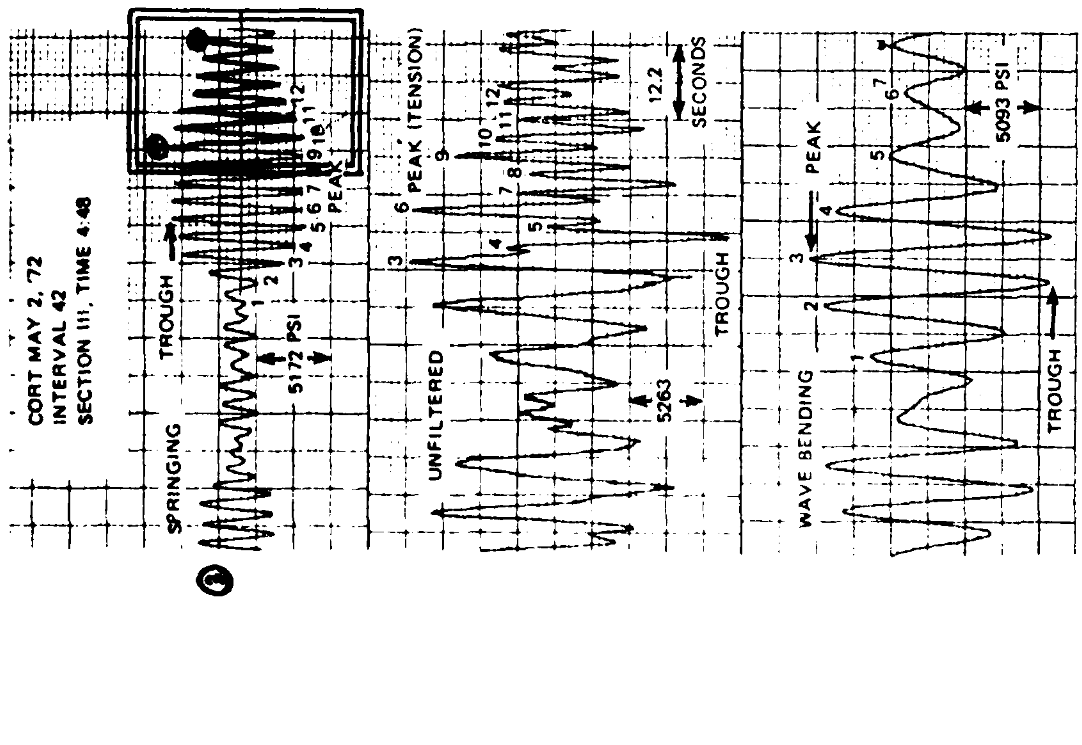
APPENDIX C

Mechanical Analysis Data

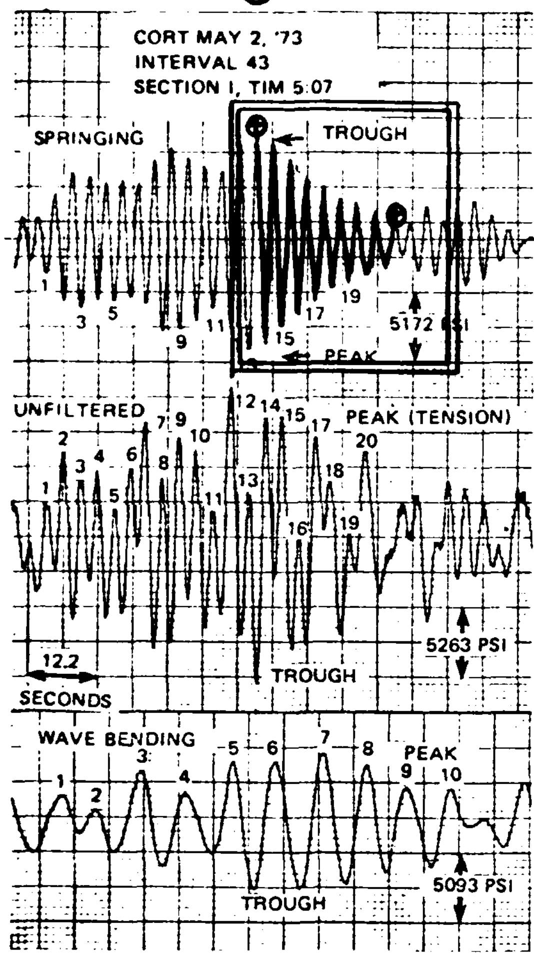

CORT MAY 2, '73
INTERVAL 42
SECTION II, TIME 4:44
SPRINGING
5172 PSI
TROUGH
PEAK
UNFILTERED
PEAK (TENSION)
TROUGH
5263 PSI
12.2 SECONDS
WAVE BENDING
PEAK
TROUGH
5093 PSI


CORT MAY 2, '73
INTERVAL 42
SECTION I, TIME 4:35
SPRINGING
5172 PSI
TROUGH
TROUGH
PEAK
UNFILTERED
PEAK (TENSION)
5263 PSI
TROUGH
12.2 SECONDS
WAVE BENDING
PEAK
TROUGH
5093 PSI


CORT MAY 2, '73
INTERVAL 42
SECTION IV, TIME 4:57
SPRINGING
TROUGH
PEAK
5172 PSI
UNFILTERED
PEAK
TROUGH
5263 PSI
12.2
SECONDS
WAVE BENDING
PEAK
TROUGH
5093

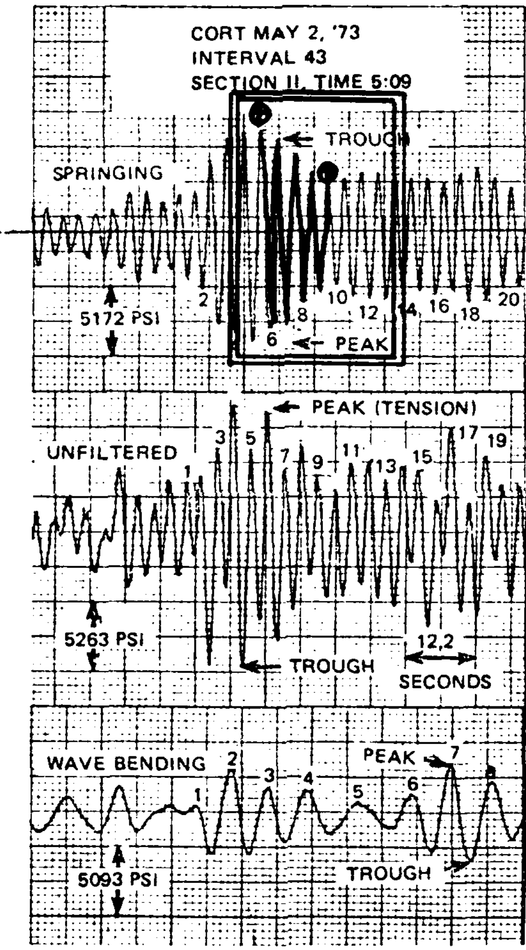

CORT MAY 2, '72
INTERVAL 42
SECTION III, TIME 4:48
SPRINGING
TROUGH
PEAK
5172 PSI
UNFILTERED
PEAK (TENSION)
TROUGH
5263
12.2
SECONDS
WAVE BENDING
PEAK
TROUGH
5093 PSI

5



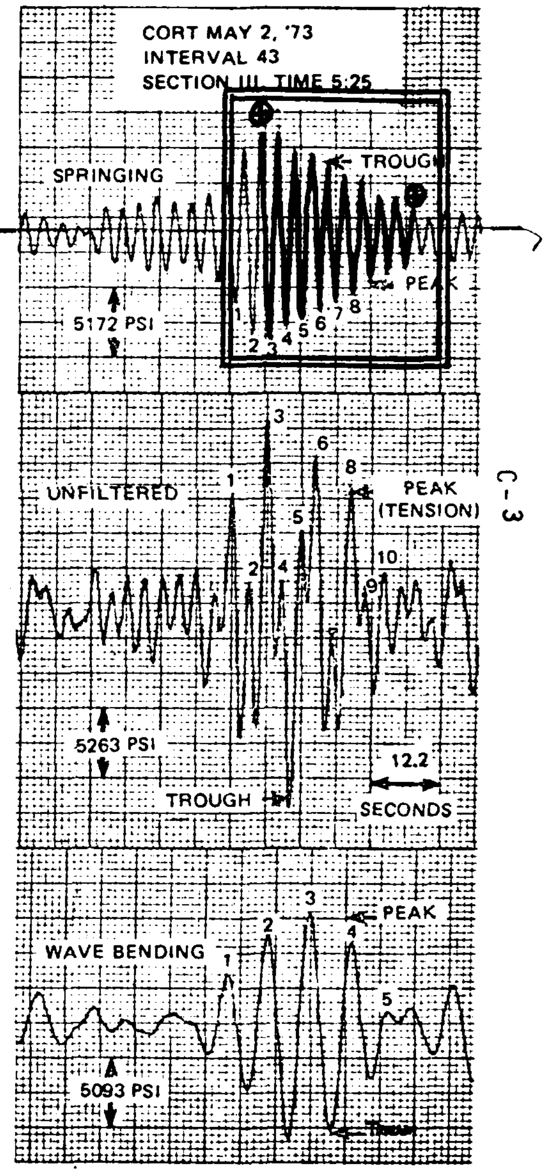
CORT MAY 2, '73
INTERVAL 43
SECTION I, TIM 5:07
SPRINGING
TROUGH
PEAK
5172 PSI
UNFILTERED
PEAK (TENSION)
TROUGH
5263 PSI
12.2
SECONDS
WAVE BENDING
PEAK
TROUGH
5093 PSI

6

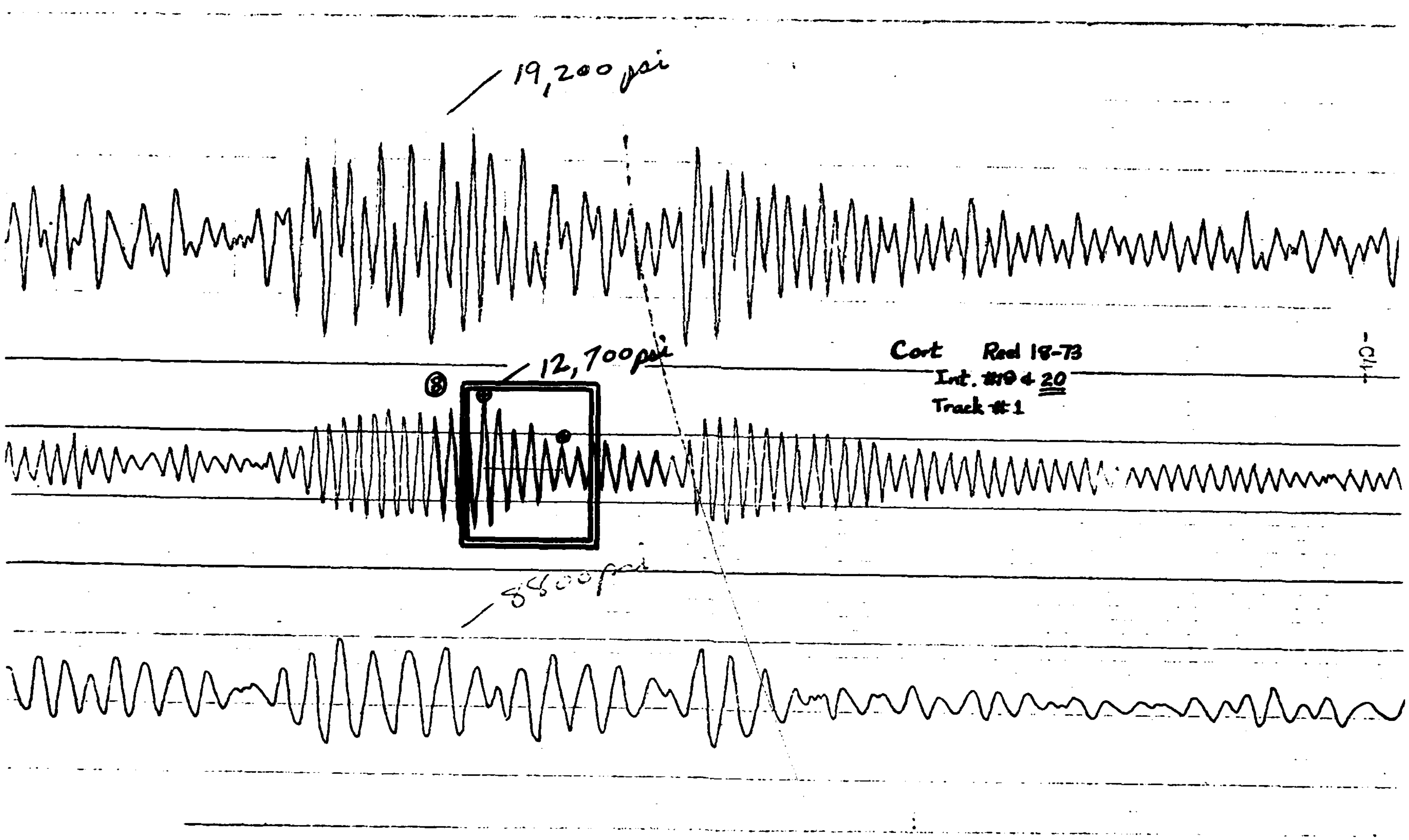


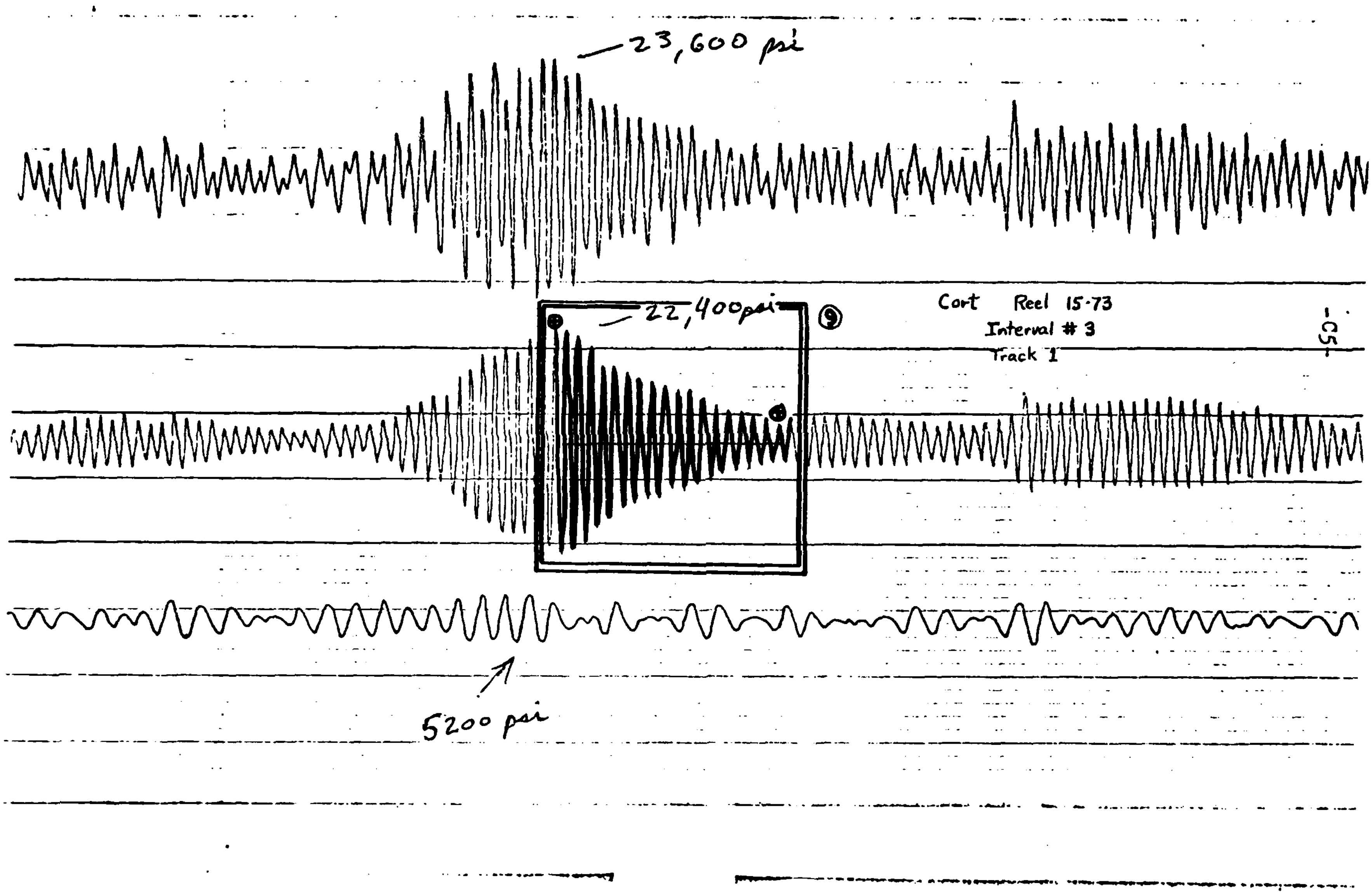
CORT MAY 2, '73
INTERVAL 43
SECTION II, TIME 5:09
SPRINGING
TROUGH
PEAK
5172 PSI
UNFILTERED
PEAK (TENSION)
TROUGH
5263 PSI
12,2
SECONDS
WAVE BENDING
PEAK
TROUGH
5093 PSI

7

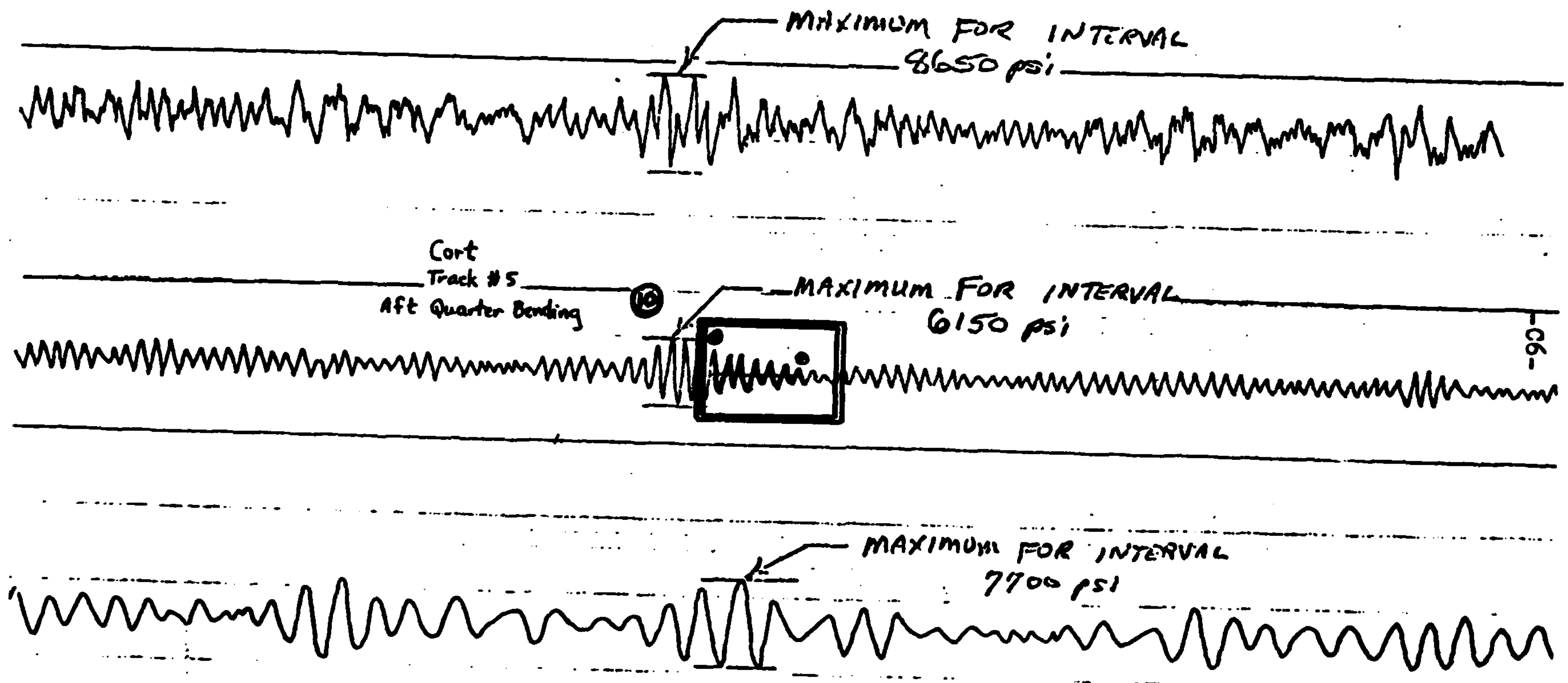


CORT MAY 2, '73
INTERVAL 43
SECTION III, TIME 5:25
SPRINGING
TROUGH
PEAK
5172 PSI
UNFILTERED
PEAK (TENSION)
TROUGH
5263 PSI
12.2
SECONDS
WAVE BENDING
PEAK
5093 PSI


19,200 psi
12,700psi
8800psi
Cort Reel 18-73
Int. #19 & 20
Track #1
-C4-



23,600 psi
22,400 psi
9
Cort Reel 15-73
Interval # 3
Track 1
C5
5200 psi


MAXIMUM FOR INTERVAL
8650 psi
Cort
Track #5
Aft Quarter Bending
10
MAXIMUM FOR INTERVAL
6150 psi
MAXIMUM FOR INTERVAL
7700 psi

-C6-

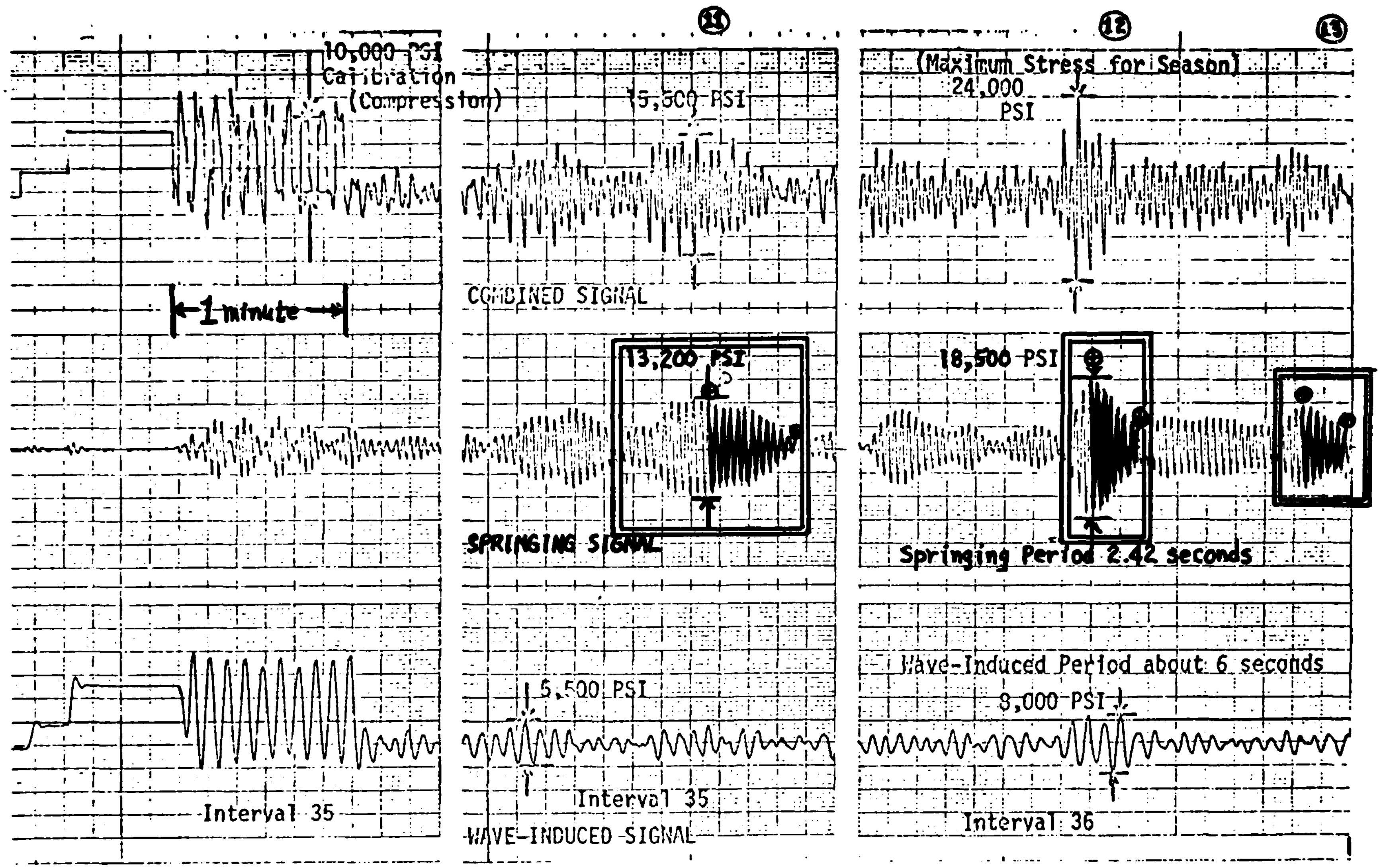
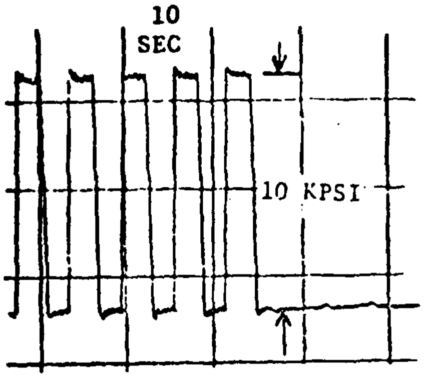


Figure 7

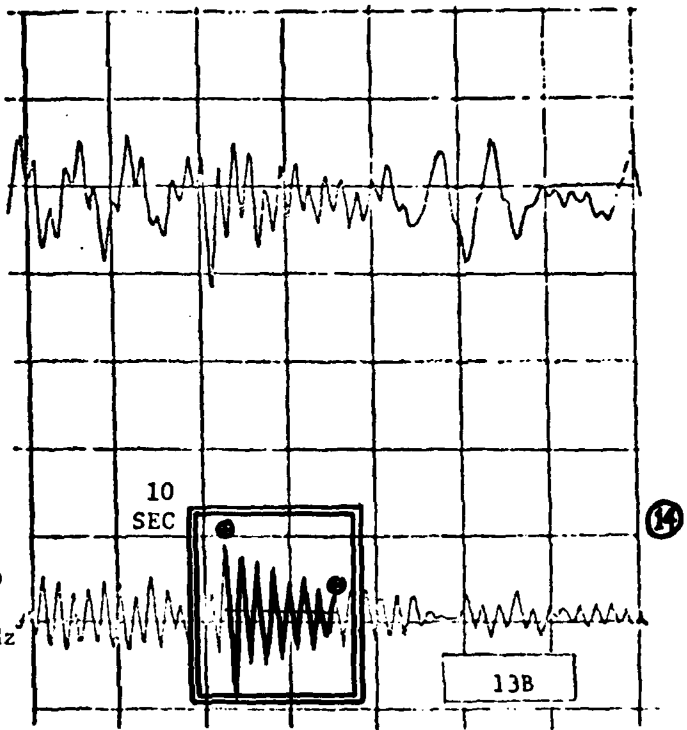
Tape 8R-74, Track 2 (Hatch 15)
Filtered Record, M/V ROGER BLOUGH



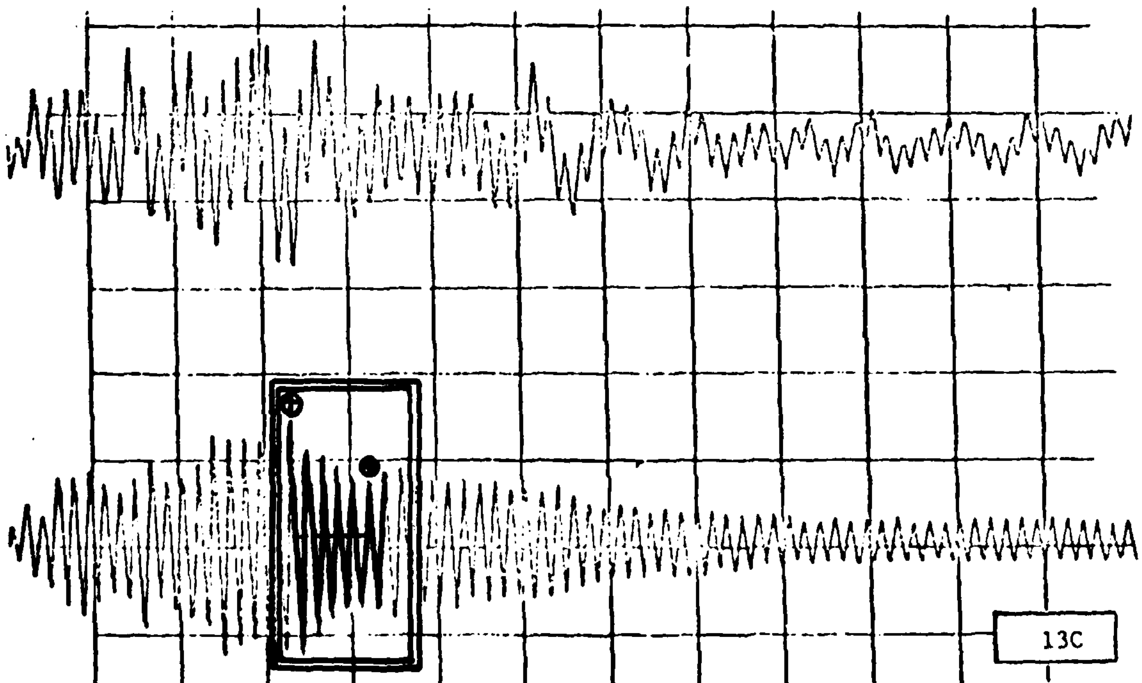
13A
CALIBRATION SIGNAL

RAW
DATA

FILTERED
DATA
0.3-0.9Hz



TIME: 1330 COURSE: 160° (65° REL.)



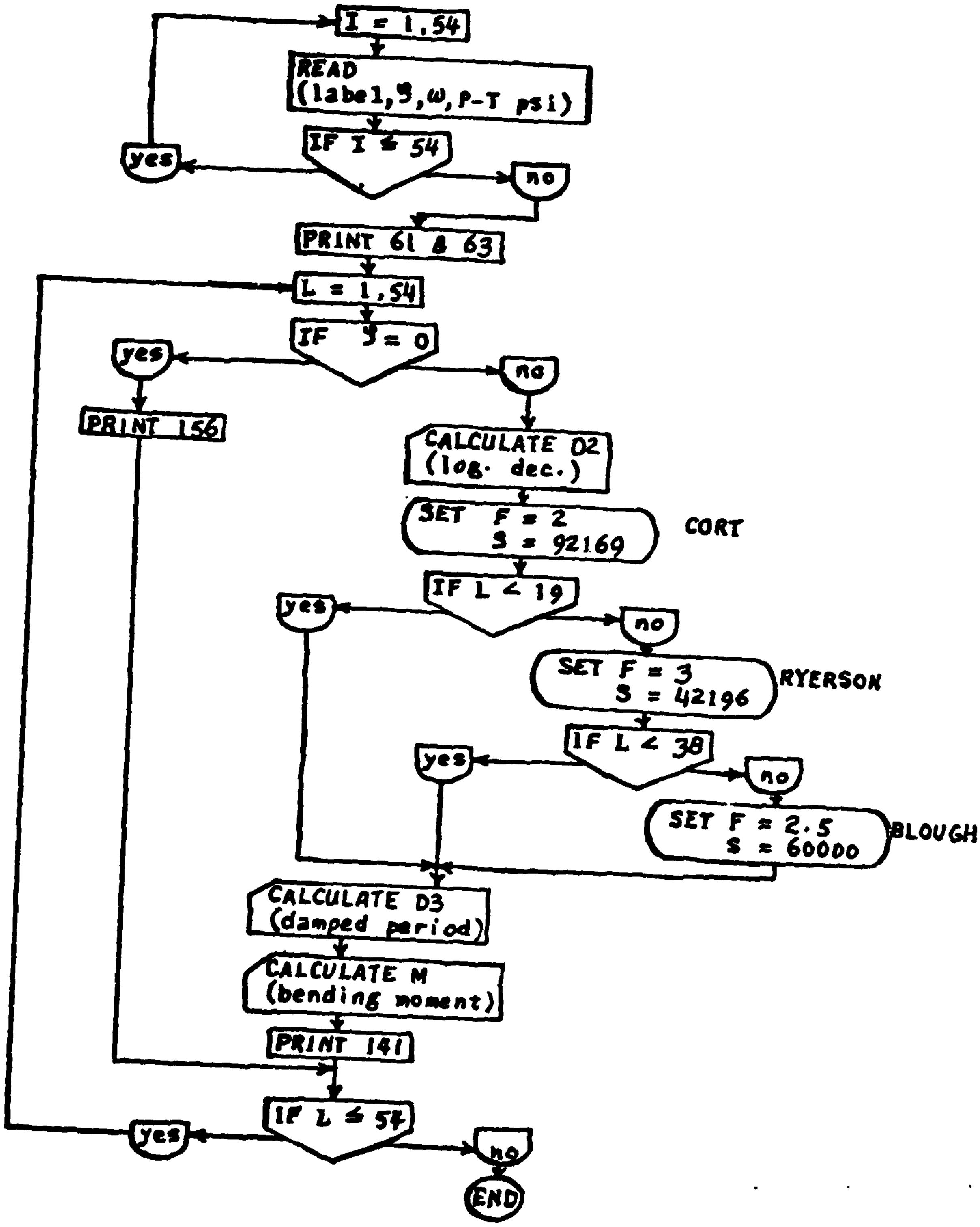
TIME: 1518 COURSE CHANGE TO 110° (115° REL.)

FIGURE 13

APPENDIX D

Basic Program plus Results

Program Flow Diagram



Program Variables

D2 : Logarithmic decrement = $= \dfrac{2 \pi \xi}{\sqrt{1 - \xi^2}}$

F : Natural frequency $= w_n =$ 2 rad/sec for CORT
= 3 rad/sec for RYERSON
= 2.5 rad/sec for BLOUGH

S : Section Modulus (in^3) = SM = 96169 for CORT
= 42196 for RYERSON
= 60000 for BLOUGH

D3 : Damped Period $= t_d = \dfrac{\delta}{w_n * \xi}$

M : Bending Moment $= BM = \dfrac{SM * \frac{\sigma_{P-T}}{2}}{12 * 2000}$

```
10 DIM D(100,4),T$64,D$(100)64
20 INPUT L$,N
30 FOR I=1 TO N
40 READ D$(I),D(I,2),D(I,3),D(I,4)
50 NEXT I
60 PRINTUSING 61
61 % LOCATOR CODE     DAMPING       FREQ      LOG. DEC  DAMPED PERIOD     B.M.
62 PRINTUSING 63
63 %                  FACTOR        (HZ)                       (SEC)           (FT-TONS)
70 FOR L=1 TO N
80 M=2
100 IF D(L,M)=0 THEN 155
110 D2=2*#PI*D(L,M)/(1-(D(L,M)↑2)↑.5)
120 F=2
121 S=92169
122 IF L<19 THEN 130
123 F=3
124 S=42196
125 IF L<38 THEN 130
126 F=2.5
127 S=60000
130 D3=D2/(F*D(L,M))
135 M=(S*(D(L,4)/2))/(2000*12)
140 PRINTUSING 141,D$(L),D(L,2),D(L,3),D2,D3,M
141 %  ##########      #.######      #.##      #.#####      ###.###        #.###↑↑↑↑
150 GOTO 160
155 PRINTUSING 156,D$(L)
156 %  ##########      --------      ----      -------      -------        ---------
160 NEXT L
170 END
180 DATA "CHSL*18*20",.018,.29,14666
190 DATA "CHSL*21*33",.0125,.33,0.,"CHSL*20*28",.0246,.43,12236,"CHSB*15*02",.01
25,.34,20306
200 DATA "CHSB*15*2E",.0181,.34,16464,"CHSB*19*24*",.0145,.3,20036,"CMSL*20*03",
.00804,.28,9150,"CMSL*20*01",0.0,0.0,0.,"CMSL*19*05",.00528,.28,9554
210 DATA "CMSB*15*30",.0167,.35,10395,"CMSB*15*06",0.0,0.0,0.,"CMSB*17*27",.0051
1,.32,9817,"CLSL*17*31",.0137,.28,3985,"CLSL*06*12",.0173,.28,3935,"CLSL*02*21",
.00719,.28,4335
220 DATA "CLSB*09*10",.0122,.31,5179,"CLSB*10*07",.0203,.31,4352,"CLSB*19*25",.0
151,.3,4595
221 DATA "RHL*5-1*1",.00423,.57,11800,"RHL*4-2*1",.00455,.56,9110,"RHB*5-1*12",.
00433,.57,13420
222 DATA "RHB*5-1*13",.00514,.57,12621,"RHB*5-1*15",.00578,.57,13960,"R[illegible]*5-1*16
",.00522,.57,11800,"RML*5-2*19",.00539,.56,7550,"RML*5-2*24",.00745,.55,7720,"RM
L*5-2*39",.00395,.57,7090
223 DATA "RMB*4-1*10",.00405,.58,6000,"RMB*4-1*11",.00362,.58,5900,"RMB*4-1*15",
.00387,.57,6540,"RMB*4-1*17",.00475,.57,7940
224 DATA "RLL*6-1*8",.00625,.57,4470,"RLL*6-1*9",.00495,.56,4390,"RLL*6-1*10",.0
0524,.44,4300
225 DATA "RLB*1-1*21",.00359,.58,4170,"RLB*1-1*24",.00335,.59,3730,"RLB*1-1*30",
0,0,0
226 DATA "BHL*13*51",0,0,0,"BHL*13*50",.0171,.383,10830,"BHB*08R*36",.0174,.42,2
0026,"BHB*13R*45",.0232,.39,16274,"BHB*08R*35",.033,.39,14226
227 DATA "BML*13*52",.0303,.382,7581,"BML*13*53",.0248,.379,5971,"BML*13*47",.01
28,.387,7090,"BMB*13*3",.0143,.41,10721,"BMB*13*5",.0227,.4,10603,"BMB*13*4",.01
2,.41,10111
228 DATA "BLL*13*46",.0202,.38,[illegible],"BLL*13*43",0,0,0,"BLL*13*45",.0103,.383,37[illegible]
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	FACTOR	(HZ)	δ	(SEC)	(FT-TONS)
CHSL*18*20	0.018000	0.29	0.11517	3.199	2.816E+04
CHSL*21*33	0.012500	0.33	0.07953	3.181	0.000E+00
CHSL*20*28	0.024600	0.43	0.15846	3.220	2.349E+04
CHSB*15*02	0.012500	0.34	0.07953	3.181	3.899E+04
CHSB*15*2E	0.018100	0.34	0.11582	3.199	3.161E+04
CHSB*19*24	0.014600	0.30	0.09309	3.188	3.847E+04
CMSL*20*03	0.008040	0.28	0.05092	3.167	1.756E+04
CMSL*20*01	--------	----	-------	-------	---------
CMSL*19*05	0.008280	0.28	0.05245	3.167	1.834E+04
CMSB*15*3D	0.016700	0.35	0.10671	3.194	2.092E+04
CMSB*15*06	--------	----	-------	-------	---------
CMSB*17*27	0.006110	0.32	0.03862	3.160	1.885E+04
CLSL*17*31	0.013700	0.28	0.08727	3.185	7.651E+03
CLSL*06*12	0.017300	0.28	0.11061	3.196	7.555E+03
CLSL*08*21	0.007190	0.28	0.04550	3.164	8.324E+03
CLSB*09*10	0.012200	0.31	0.07760	3.180	9.944E+03
CLSB*10*07	0.020300	0.31	0.13019	3.206	8.414E+03
CLSB*19*25	0.015100	0.30	0.09633	3.189	8.823E+03
RHL*5-1*1	0.004830	0.57	0.03049	2.104	1.037E+04
RHL*4-2*1	0.004550	0.56	0.02871	2.103	8.008E+03
RHB*5-1*12	0.004330	0.57	0.02732	2.103	1.179E+04
RHB*5-1*13	0.005140	0.57	0.03246	2.105	1.109E+04
RHB*5-1*15	0.005780	0.57	0.03652	2.106	1.227E+04
RHB*5-1*16	0.005220	0.57	0.03297	2.105	1.037E+04
RML*5-2*19	0.005390	0.56	0.03404	2.105	6.637E+03
RML*5-2*24	0.007450	0.56	0.04716	2.110	6.795E+03
RML*5-2*39	0.003950	0.57	0.02491	2.102	6.232E+03
RMB*4-1*10	0.004050	0.58	0.02555	2.102	5.274E+03
RMB*4-1*11	0.003620	0.58	0.02282	2.102	5.186E+03
RMB*4-1*15	0.003870	0.57	0.02441	2.102	5.749E+03
RMB*4-1*17	0.004750	0.57	0.02998	2.104	6.979E+03
RLL*6-1*8	0.006250	0.57	0.03951	2.107	3.929E+03
RLL*6-1*9	0.004950	0.56	0.03125	2.104	3.859E+03
RLL*6-1*10	0.005240	0.44	0.03309	2.105	3.780E+03
RLB*1-1*21	0.003590	0.58	0.02263	2.101	3.665E+03
RLB*1-1*24	0.003360	0.59	0.02118	2.101	3.322E+03
RLB*1-1*90	--------	----	-------	-------	---------
BHL*13*51	--------	----	-------	-------	---------
BHL*13*50	0.017100	0.38	0.10931	2.556	1.353E+04
BHB*08R*36	0.017400	0.42	0.11126	2.557	2.503E+04
BHB*13R*45	0.023200	0.39	0.14923	2.572	2.034E+04
BHB*08R*35	0.033000	0.39	0.21442	2.599	1.778E+04
BML*13*52	0.030300	0.38	0.19632	2.591	9.476E+03
BML*13*53	0.024800	0.37	0.15978	2.577	7.463E+03
BML*13*47	0.012800	0.38	0.08146	2.545	8.862E+03
BMB*13*3	0.014300	0.41	0.09115	2.549	1.340E+04
BMB*13*5	0.022700	0.40	0.14594	2.571	1.325E+04
BMB*13*4	0.012000	0.41	0.07631	2.543	1.263E+04
BLL*13*46	0.020200	0.38	0.12953	2.565	5.573E+03
BLL*13*43	--------	----	-------	-------	---------
BLL*13*45	0.010300	0.38	0.06539	2.539	4.721E+03
BLB*8R*28	--------	----	-------	-------	---------
BLB*8R*38	0.011700	0.43	0.07438	2.543	5.052E+03
BLB*13R*57	0.018400	0.42	0.11777	2.560	5.138E+03

APPENDIX E

American Bureau of Shipping letter

American Bureau of Shipping
Sixty-five Broadway
New York, N. Y. 10006

Refer to YNC/bv
File Ref.

23 January 1979

Lt. Mark Noll
Department of Transportation
U.S.Coast Guard (G-DSA-1/TP44)
Washington, D.C. 20590

Dear Mark:

We have received the package containing a sample copy for the purpose of transfer of fund and your report "Evaluation of SDRC Damping Analysis" dated 18 October 1978. Matters regarding the feasibility study are now being handled by Dr. D. Liu, our Chief Engineer. I believe he has discussed this matter with you on the phone already.

I would like to offer some comments, as you so requested, on the SDRC work and your report as follows. I believe that the SDRC work and our efforts (hereby abbrieviation as ABS' work for easy reference) on analyzing the damping coefficient through the use of sea trial data (stress history) are based on the same principle, a happy coincidence. There are, however, major differences as noted below:

(1) SDRC uses a multi-degree-of-freedom system and ABS uses a one d.o.f. system. It is believed that the inter-modal correlation in the power spectrum is neglibible with the provision of (2) as a safeguard.

(2) We employ partial spectra based on a concept proposed by Vanmarcke (OTC 1971) rather than the full spectrum.

(3) ABS' work also incorporate the equilibrium branch of a wave spectrum which is assumed to be inversely proportional to the fifth power of frequency. This is also a departure from Vanmarcke's original idea.

(4) Since the power spectrum transformed from the stress time-history is expressed as blocks with constant ordinate within a frequency cell (bin), the fabricated spectrum should also be in the same form as in the case in our analysis. Our matching criterion, as proposed by Vanmarcke, is to have the quantity p of the fabricated spectrum to match that of the measured spectrum where

$$p = 1 - m_1^2/(m_0 m_2)$$

TELEPHONE 212-440-0300 CABLE ADDRESS "RECORD" TWX 710 581 3089 TELEX ITT 421966 RCA 232099 WUI 620353

TO Lt. Mark Noll DATE 1/23/79 FILE REF
Department of Transportation
U.S. Coast Guard

and

$$m_i = \text{ith spectral moment.}$$

In pursuing the curve fitting, the damping coefficient as well as the resonance frequency (not necessarily equal to the central value of the frequency cell) are regarded as variables. In doing so, the two spectra will have the same zero, first and second spectral moments.

We have employed 542 intervals of the "Cort" data set (1973 season) from which one value of the damping coefficient is obtained for each interval and the resulting histogram is shown in Fig. 1.

It should be noted that not all results are reliable. Some intervals have spectral ordinates so small that they are nothing but small ripples in the power spectrum (no springing). For this reason, cutoff values for the RMS value (square root of m_0) have been used and the resulting mean and standard deviation of the set of results are shown as functions of this cutoff value as displayed in Fig. 2. As a result, those unreliable intervals are screened out as the cutoff RMS value increases and both the mean and standard deviation exhibit stablized trend. In this figure, the curves labeled as "$C \leq 0.039$" represent further screening of elimininating the unusually high values of the damping coefficients which amounts to less than 5% of all intervals computed. It is evident that the damping coefficient is virtually independent of the stress level as the "stablized" portion of the curves indicate. It suffices to apply this conclusion without limiting it to the high stress samples since only the cases of high springing stress are of interest.

It is apparent although the number of intervals considered in the SDRC calculation is very small compared with our sample size, that the mean value of their results as well as the spread agree very well with our results (i.e., mean ≈ 0.0145, standard deviation ≈ 0.0045).

As indicated in your report, results obtained utilizing the logarithmic decrement method are much higher than those from spectral synthesis. We have observed the same tendency through the case of the RANDOMDEC method which yields a vibration signature in the time domain. Such time domain signatures are subsequently analyzed by way of logarithmic decrement. One possible explanation of the discrepancy between results obtained in these two methods is that in the time domain, the signal is not truly monochromatic. In this regard, perhaps

TO Lt. Mark Noll
Department of Transportation
U.S. Coast Guard

DATE 1/23/79 FILE REF

the test of a ship with zero surge would yield closer agreement between the two approaches.

Based on the discussion presented above, it appears that the spectral synthesis approach is more promising. In this regard, the approximate shape of the wave energy spectrum should also be accounted for as our analysis indicates that the omission of this aspect leads to about 20% higher values for the damping coefficient on the average.

I hope the preceding comments would provoke more subsequent discussions on this subject. If you have any suggestions or comments, please feel free to contact me.

Sincerely yours,

Y. N. Chen

Y.N. Chen

Encl.

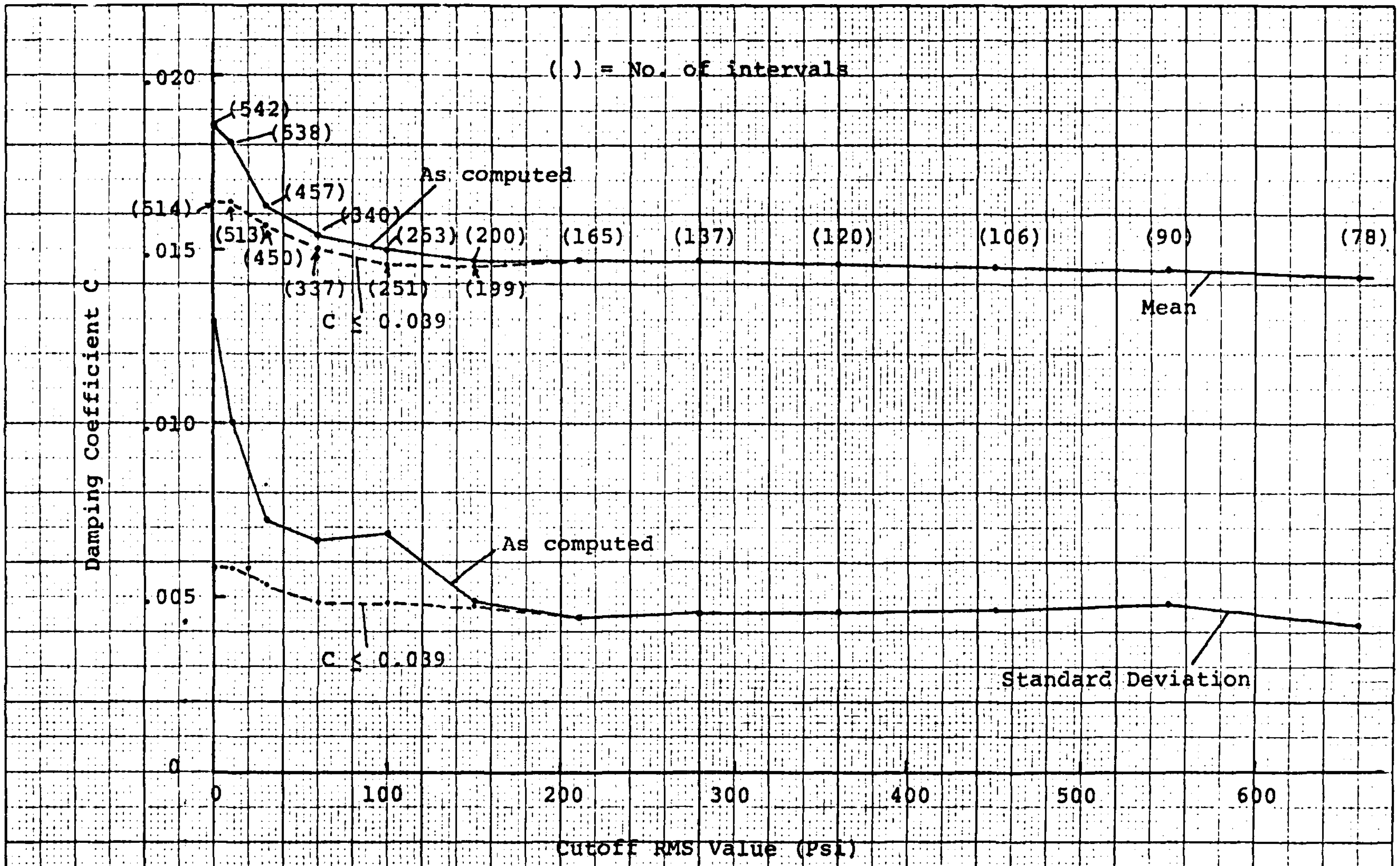

Fig. 2 Variation of C Versus Cutoff RMS Value

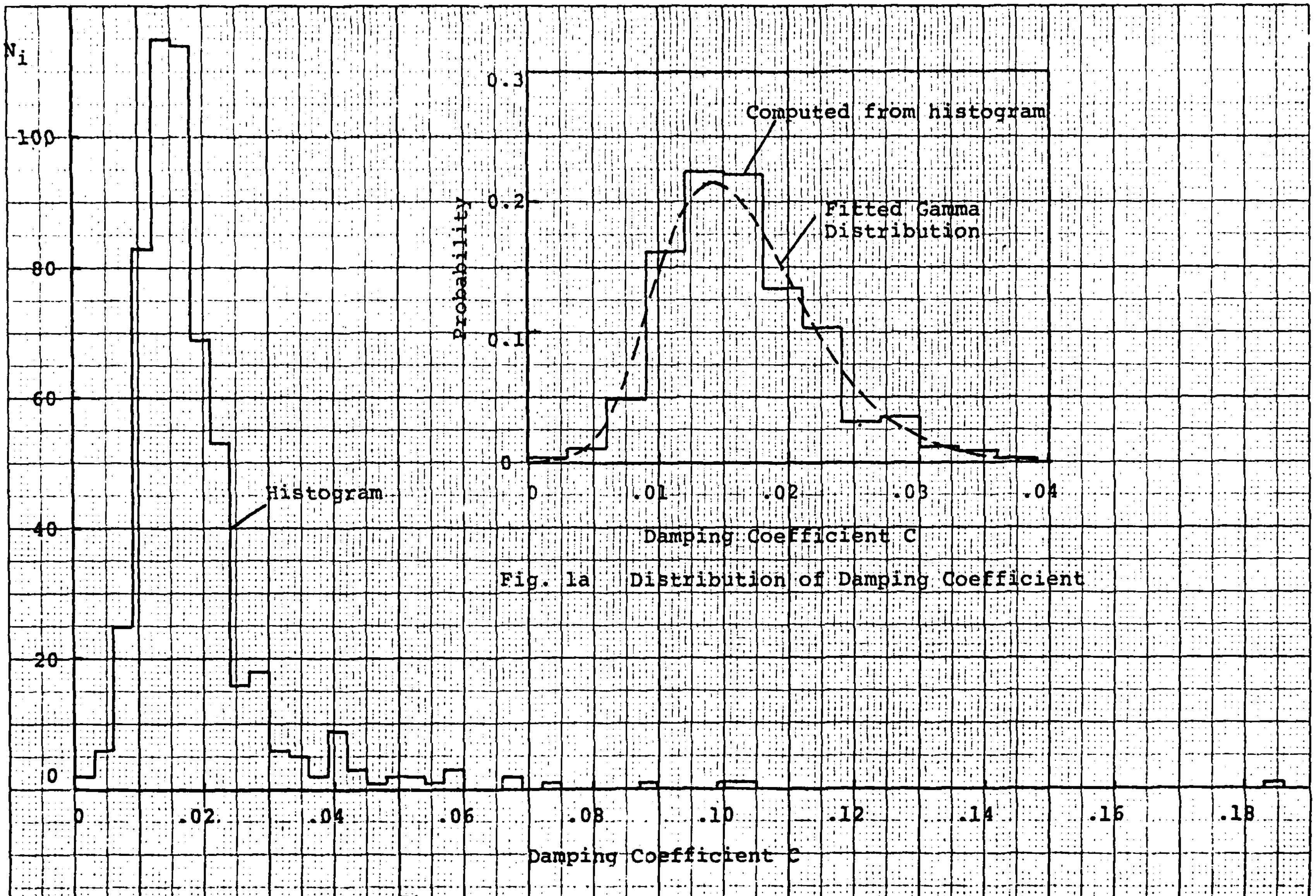

Fig. 1a Distribution of Damping Coefficient

Fig. 1 Histograms of Damping Coefficient

APPENDIX F

Power Spectral Densities submitted
by SDRC

	Tape No.	SDRC Reference ID	Record No.	Track No.	Frequency (Hz)	% Critical Damping	USCG Interval	Number of Averages In PSD	Tape Count (ft.)
	1	1 2	0	11	—	—	1	400	160-390
	1	2 2	1	3	.34	.0125	2	100	188-238
CORT	1	3 2	2	3	.34	.0181	2E	100	443-493
	1	4 2	3	3	.35	.0167	3D	100	695-755
	1	5 2	4	11	.28	.00804	3	100	512-640
	1	6 2	5	10	.28	.0083	5	100	640-700
	1	7 2	6	3	—	?	6	60	850-900
	1	8 2	7	9	.31	.0203	7	100	2360-2513
	1	9 2	8	8	.31	.0122	10	50	975-1044
	1	10 2	9	6	.28	.0173	12	50	632-689
	1	11 2	10	4	.29	.0180	20	400	1245-1585
	1	12 2	11	7	.28	.00719	21	50	1130-1185
	1	13 2	12	10	.30	.0147	24	400	2130-2498
	1	14 2	13	10	.30	.0151	25	50	2500-2567
	1	15 2	14	5	.32	.00611	27	50	1338-1388
	1	16 2	15	1			28	50	1820-1907
	1	17 2	16	12	.43	.0246	28	50	1350-1400
	1	18 2	17	5	.32	.0137	31	50	1566-1615
	1	19 2	18	2	.33	.0125	33	400	3177-3534
BLOUGH	1	20 2	19	12	.39	.033	35	50	1699-1748
	1	21 2	20	12	.42	.0174	36	50	1748-1798
	1	22 2	21	12	.43	.0117	38	50	1848-1898
	1	23 2	22	14	.39	.0232	45	50	2395-2444
	1	24 2	23	14	.42	.0184	57	50	3044-3093
RYERSON	2	25 2	24	4	.57	.00483	1	100	0-100
	2	26 2	25	5	.56	.00539	1	300	26-315
BLOUGH	2	27 2	26	7	.41	.0143	3	50	150-196
	2	28 2	27	7	.41	.0120	4	50	200-248
	2	29 2	28	7	.40	.0227	5	50	250-292
RYERSON	2	30 2	29	5	.56	.00745	6	200	515-650
	2	31 2	30	3	.56	.00455	7	100	570-630
	2	32 2	31	6	.57	.00625	15	50	1270-1310
	2	33 2	32	6	.56	.00495	16	50	1320-1360
	2	34 2	33	6	.44	.00524	17	50	1370-1410
	2	35 2	34	2	.58	.00405	18	100	1650-1705
	2	36 2	35	2	.58	.00362	19	300	1715-1840
	2	37 2	36	4	.57	.00433	26	400	1843-1995
	2	38 2	37	4	.57	.00514	27	400	2200-2375

RYERSON	2	39	2	38	4	.57	.00578	28	400	2570-2725
	2	40	2	39	4	.57	.00422	29	100	2930-2993
	2	41	2	40	2	.57	.00387	30	100	2535-2585
	2	42	2	41	2	.57	.00475	32	200	2670-2755
	2	43	2	42	1	.58	.00359	35	50	1905-1945
	2	44	2	43	5	—	—	39	200	3170-3255
	2	45	2	44	5	.57	.00395	40	200	3320-3410
	2	46	2	45	1	.59	.003365	40	50	2140-2200
BLOUGH	2	47	2	46	7	—	—	43	50	2240-2275
	2	48	2	47	7	.383	.0103	45	50	2338-2373
	2	49	2	48	7	.380	.0202	46	50	2395-2425
	2	50	2	49	7	.387	.0128	47	50	2440-2475
	2	51	2	50	7	.383	.0171	50	50	2590-2625
	2	52	2	51	7	—	—	51	50	2640-2680
	2	53	2	52	7	.382	.0303	52	50	2690-2730
	2	54	2	53	7	.379	.0248	53	50	2740-2780

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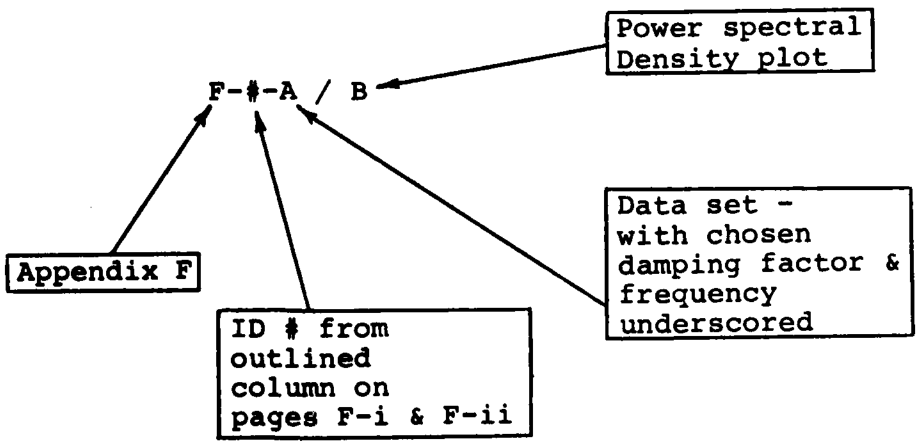
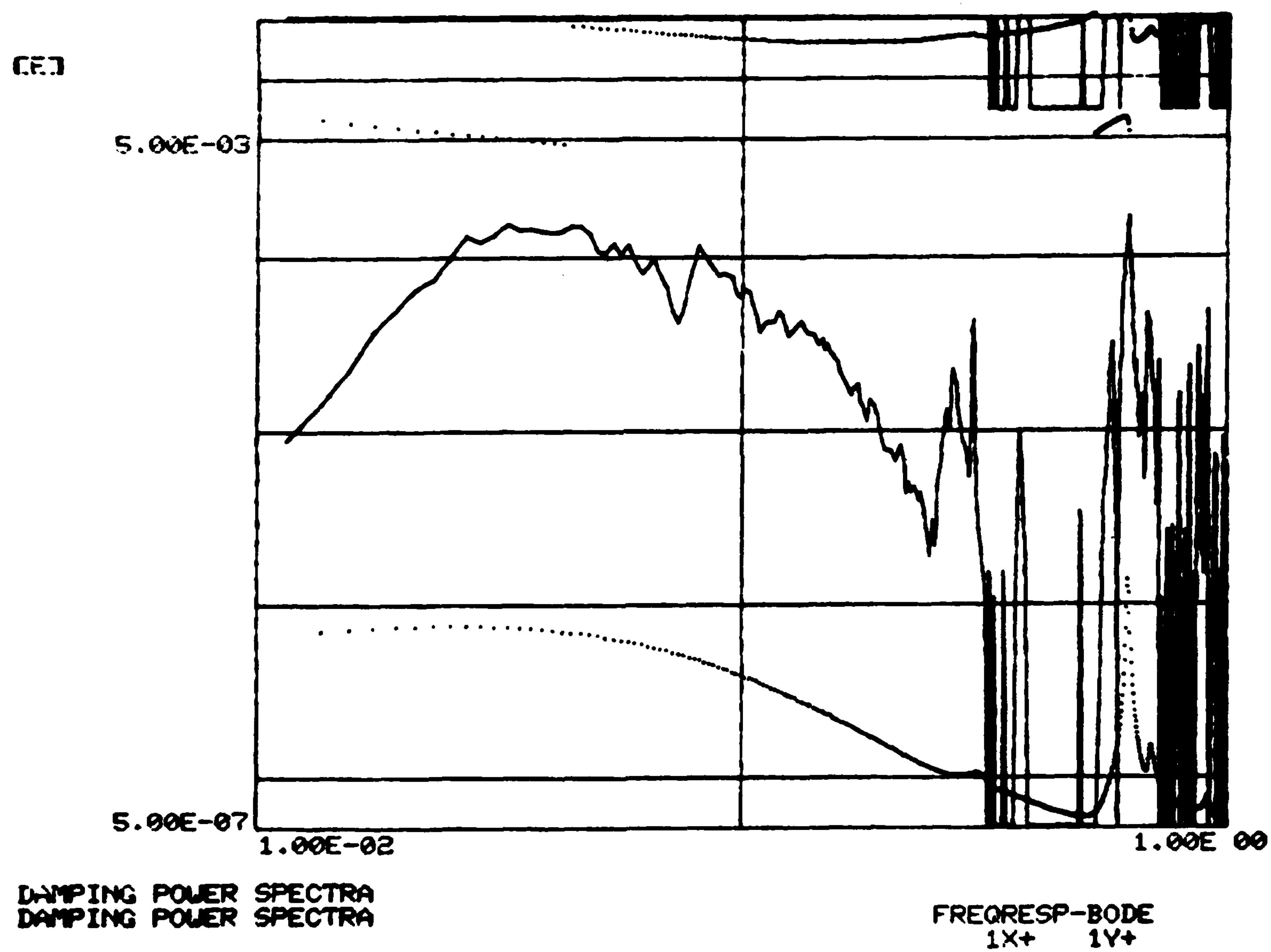
for following pages:



CORT

ESTIMATED ROOTS (1X+ 1Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1408E-01	1.000	0.1960E-06	3.142
2	0.8275E-01	0.9604	0.8257E-06	-1.343
3	0.1234	0.6269	0.1391E-11	1.369
4	0.2948	0.1749	0.6609E-08	0.5438
5	0.3162	0.3521E-01	0.1350E-08	-2.041
6	0.5606	0.5288E-01	0.1624E-08	2.436
7	0.6266	0.5397E-02	0.3414E-07	-0.6885E-01
8	0.6972	0.1251E-01	0.4270E-08	-0.2174
9	0.8708	0.6282E-01	0.9081E-12	-0.2211
10	0.9171	0.1026E-01	0.1213E-08	-0.6822
11	1.058	0.3262	0.1541E-07	0.1210E-05



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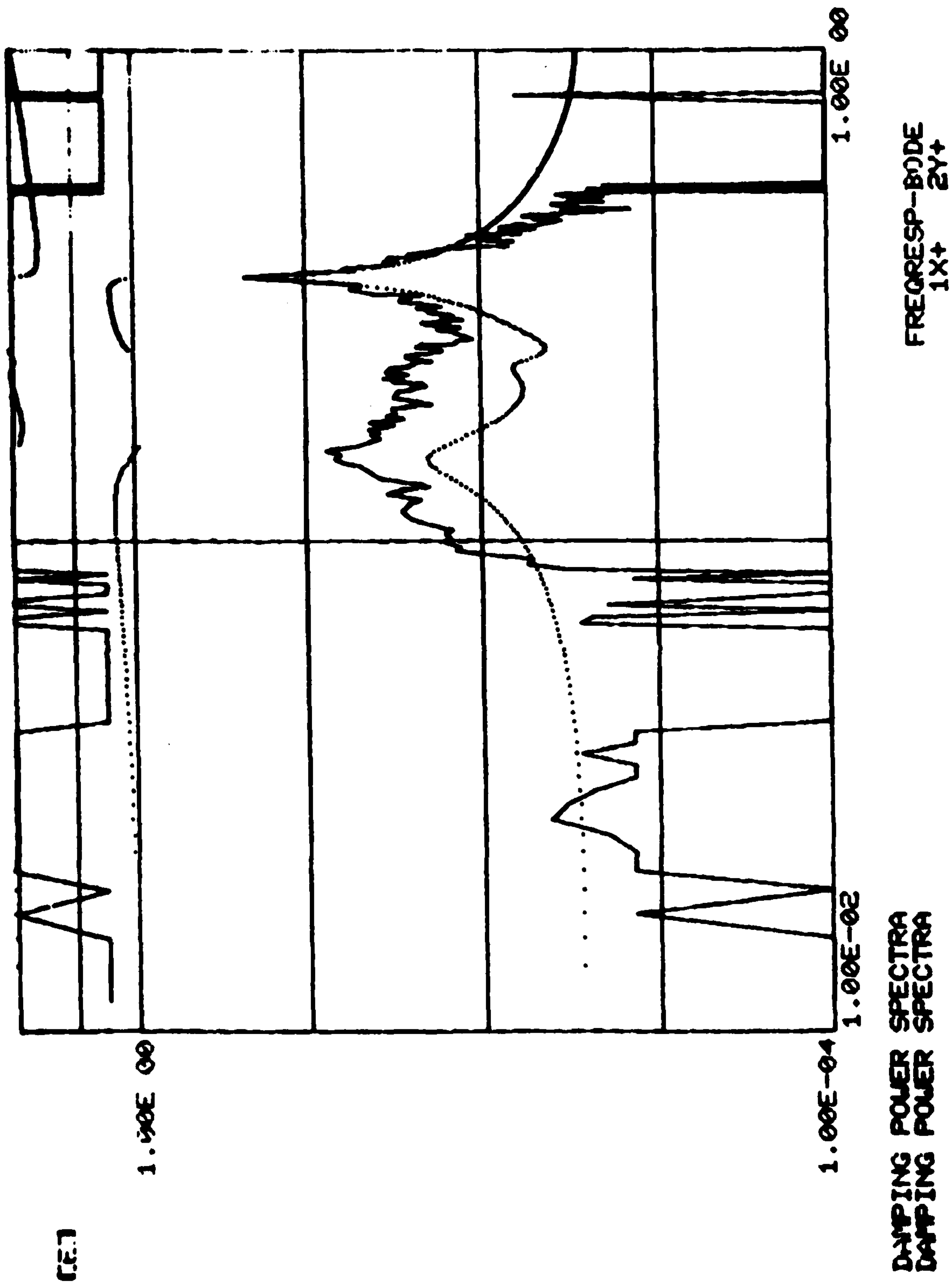
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 1Y+

CORT

ESTIMATED ROOTS (1X+ 2Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.5261E-02	1.000	0.9041E-06	-0.8724E-02
2	0.2469	1.000	0.1975E-08	2.902
3	0.1155	0.1981	0.1760E-05	-1.050
4	0.1476	0.8880E-01	0.1677E-03	0.2385
5	0.2305	0.5714E-01	0.2644E-04	-0.6653
6	0.3455	0.1247E-01	0.4804E-03	-0.2004E-01
7	0.3948	0.2279E-01	0.5835E-05	-1.366
8	0.4562	0.5420E-01	0.1298E-05	-1.783
9	0.5068	0.3782E-01	0.4574E-06	-0.9194
10	0.7963	0.3452E-01	0.2013E-09	1.323
11	0.8109	0.4167E-03	0.1914E-06	-0.6590E-02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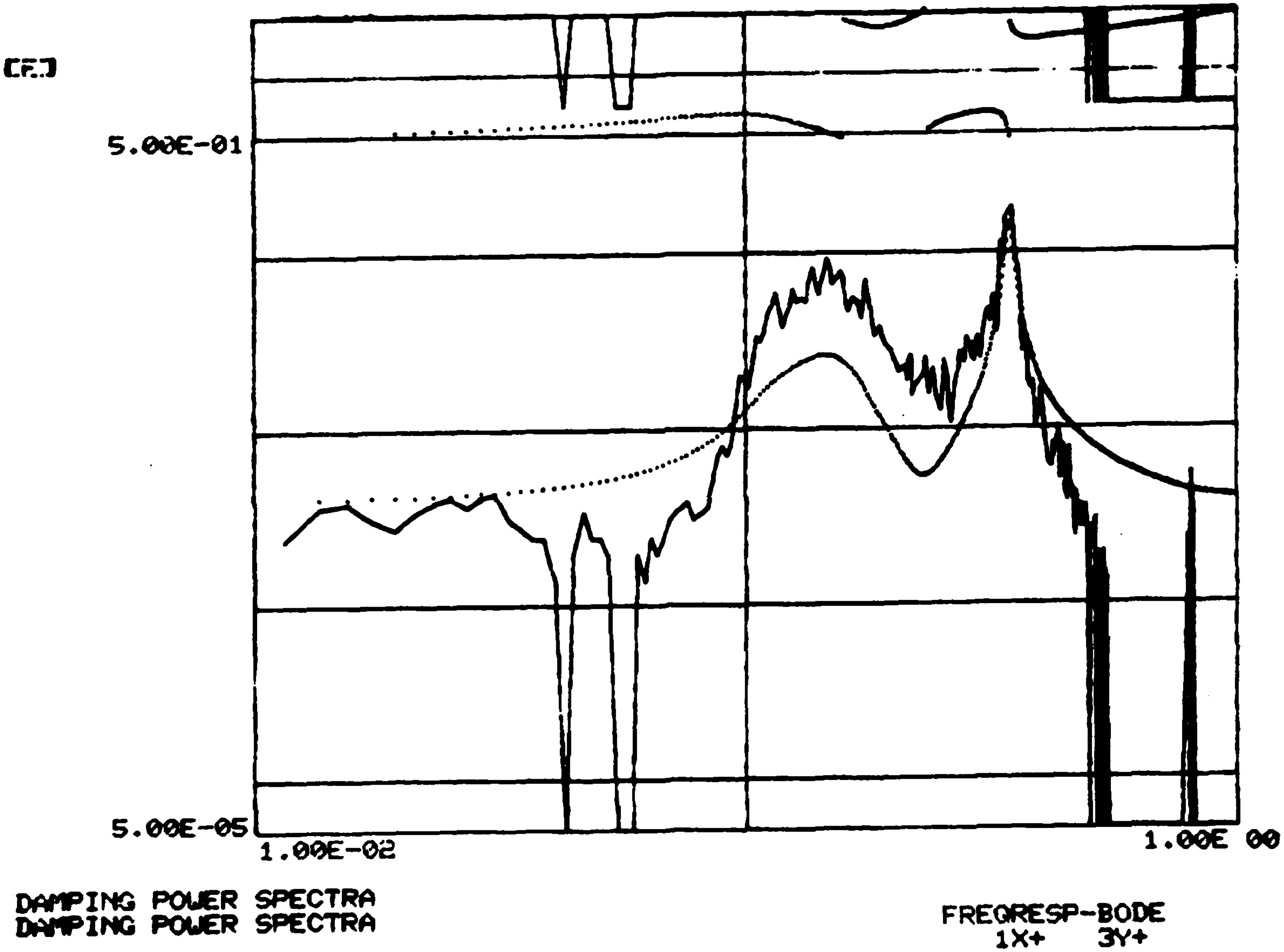
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+
2Y+

CORT

ESTIMATED ROOTS (1X+ 3Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.4071E-01	1.000	0.2670E-07	2.894
2	0.1177	0.2249	0.1722E-03	1.824
3	0.1566	0.1742	0.4183E-03	-0.4368
4	0.2750	0.2842	0.7068E-04	2.038
5	0.3449	0.1808E-01	0.6887E-03	-0.8678E-01
6	0.3941	0.2614E-01	0.6661E-05	2.614
7	0.4865	0.5840E-01	0.8692E-08	-2.348
8	0.5321	0.1046	0.5548E-06	-3.124
9	0.7932	0.5204E-02	0.1463E-07	0.2672
10	0.8103	0.1316E-02	0.2388E-06	-0.2747E-01
11	1.167	0.5159	0.1230E-11	-0.7412E-08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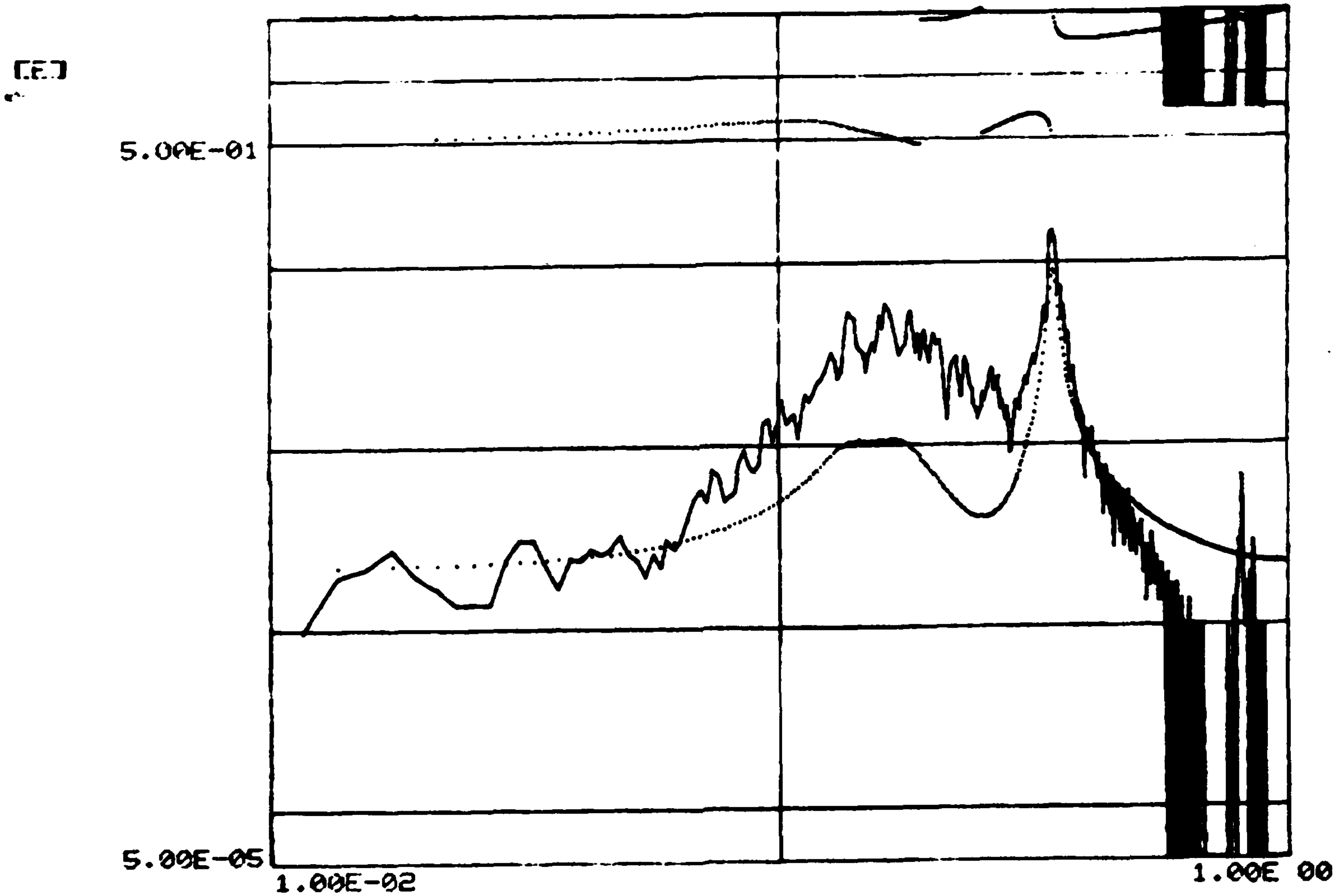
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 3Y+

CORT

ESTIMATED ROOTS (1X+ 4Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.5855E-01	1.000	0.3743E-05	-0.1486E-03
2	0.1394	0.1426	0.6993E-04	0.5661
3	0.1751	0.1504	0.1235E-03	0.5750E-01
4	0.3263	0.1501	0.6330E-04	-2.317
5	0.3460	0.1672E-01	0.3899E-03	0.2255E-01
6	0.4611	0.1476	0.3281E-05	0.4132
7	0.4987	0.1186E-01	0.2943E-07	-0.8076
8	0.6162	0.7756E-01	0.5355E-07	2.943
9	0.8131	0.5461E-03	0.2923E-06	-0.5594
10	0.8280	0.1278E-01	0.2605E-06	0.6978
11	1.004	0.8866E-01	0.4449E-07	0.5214E-03



DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 4Y+

CORT

ESTIMATED ROOTS (1X+ 5Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1273	1.000	0.7518E-11	-0.1070
2	0.6881E-01	0.3200E-01	0.4335E-08	0.3104
3	0.2482	0.9224E-01	0.7928E-05	1.473
4	0.2785	0.8037E-02	0.1077E-03	-0.6889E-01
5	0.3271	0.4792E-01	0.5440E-06	-1.482
6	0.4938	0.1647	0.1848E-10	1.884
7	0.5108	0.7253E-02	0.1318E-06	-0.5920E-01
8	0.6475	0.6007E-02	0.9064E-06	-0.3801E-01
9	0.7005	0.4649E-01	0.2212E-07	2.349
10	0.9749	0.1172E-01	0.2382E-07	0.7229
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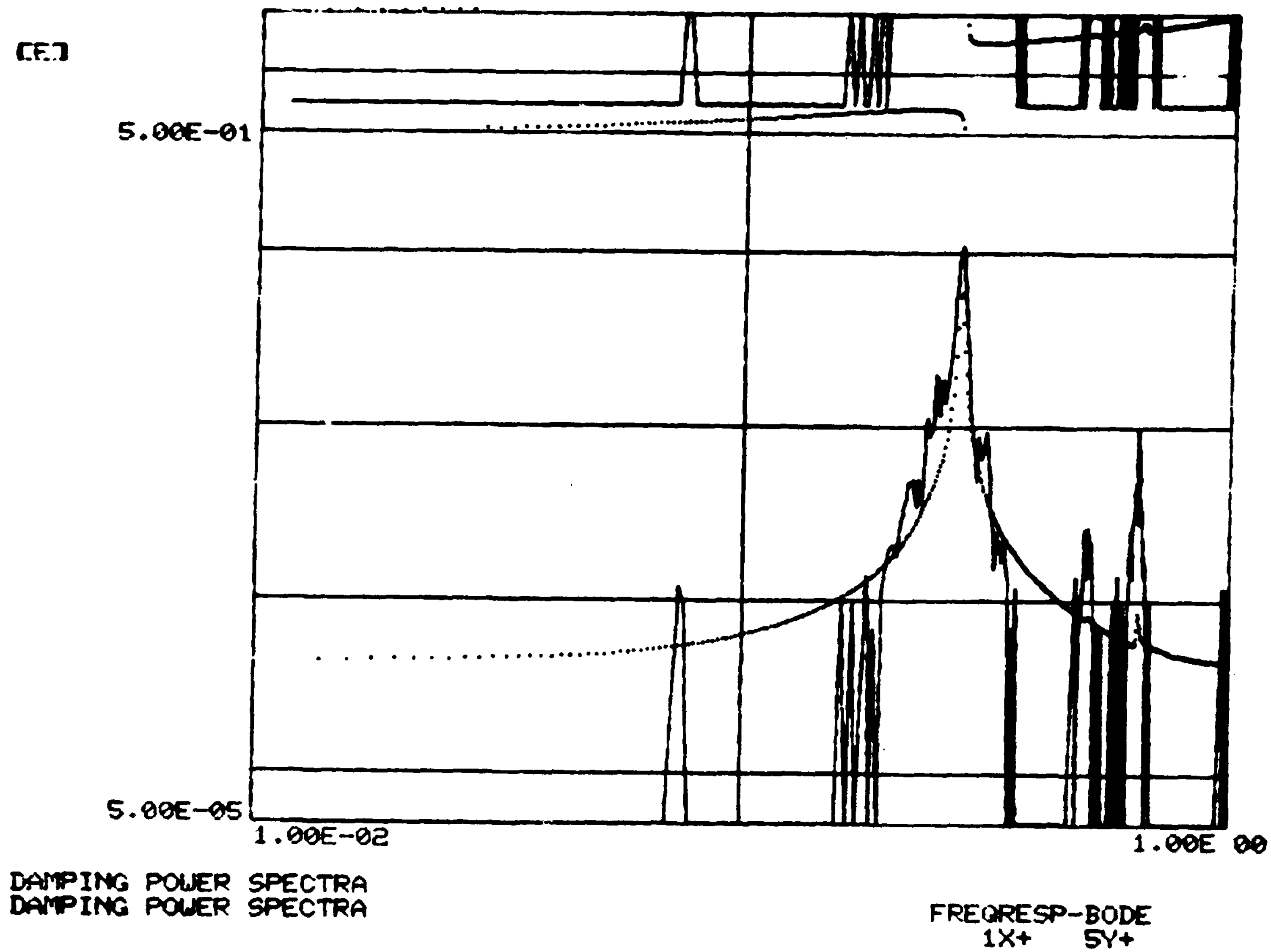

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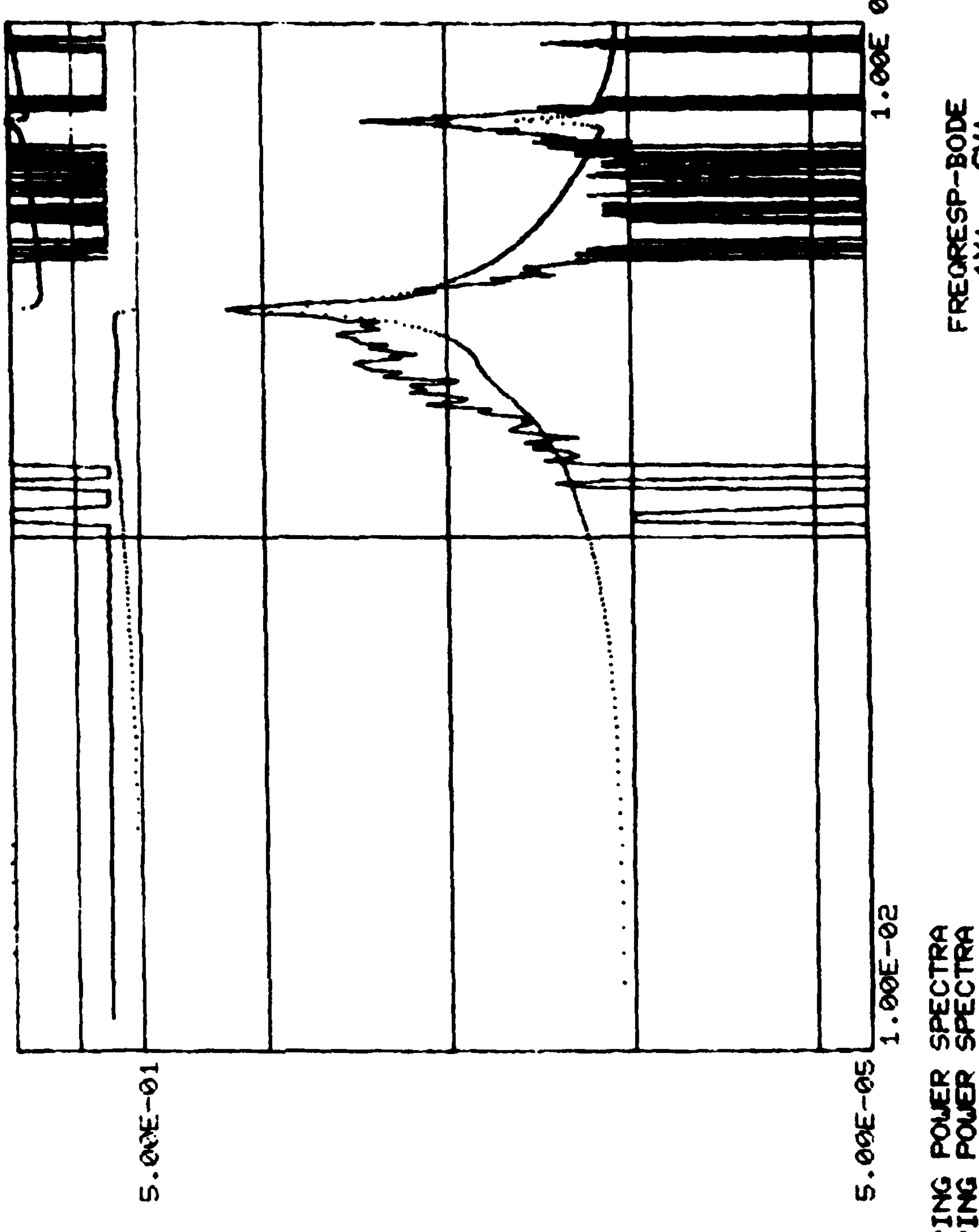
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 5Y+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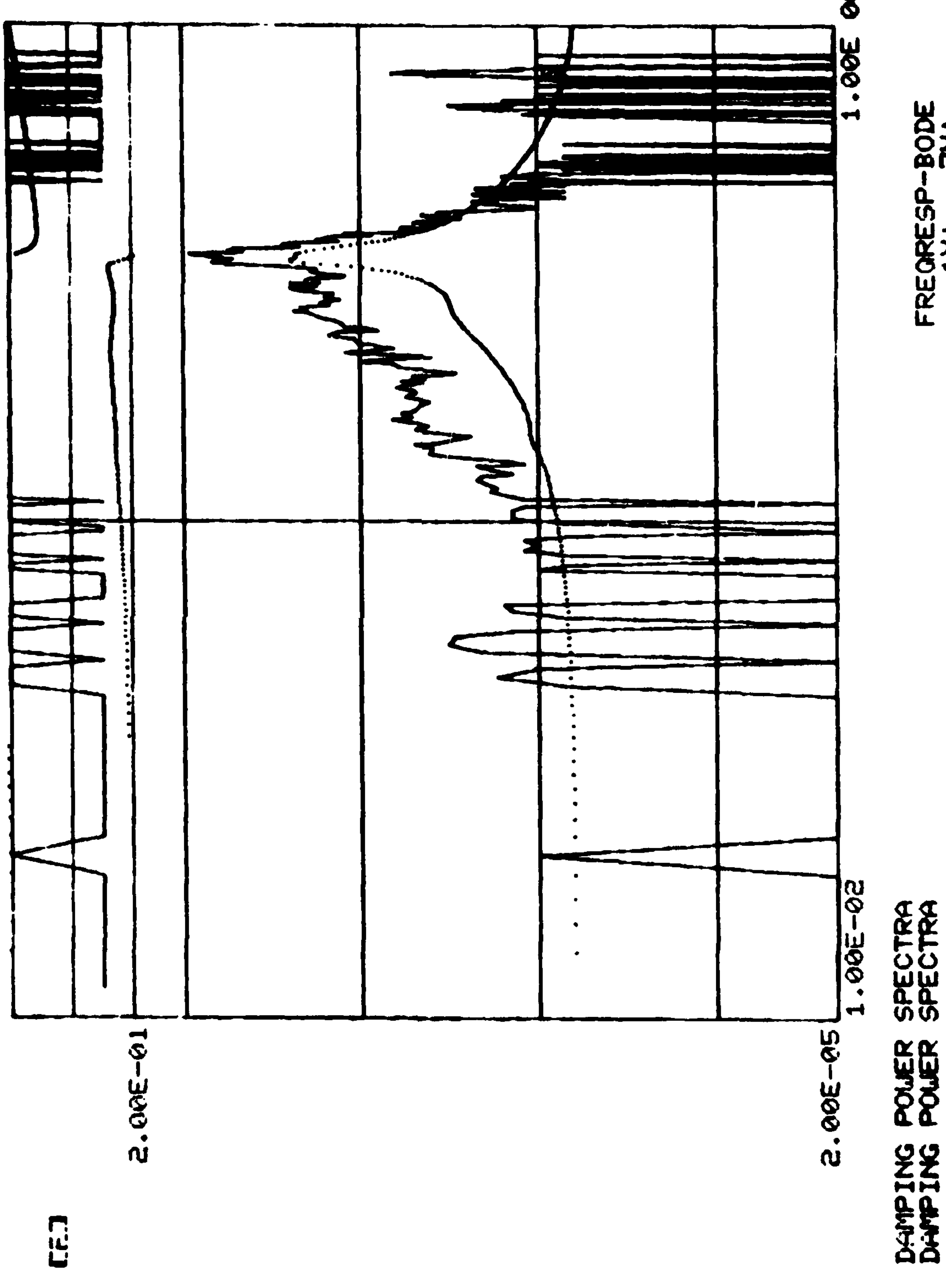
POWER SPECTRA
POWER SPECTRA
FREQRESP-BODE
1X+ 6Y+

CORT

ESTIMATED ROOTS (1X+ 7Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.4523E-01	0.5568	0.9564E-08	1.337
2	0.1466	0.6536E-01	0.5860E-06	0.7982
3	0.2664	0.1342	0.2799E-04	0.5627
4	0.3341	0.1401E-01	0.5293E-04	0.8424
5	0.3464	0.2487E-01	0.1063E-03	-0.5375
6	0.4868	0.1122	0.1547E-06	-1.224
7	0.6880	0.1049E-01	0.1503E-06	0.1196
8	0.7188	0.1061	0.1015E-06	-2.659
9	0.8037	0.3155E-02	0.4611E-06	-0.2077E-01
10	0.8679	0.4042E-02	0.3093E-08	0.6746



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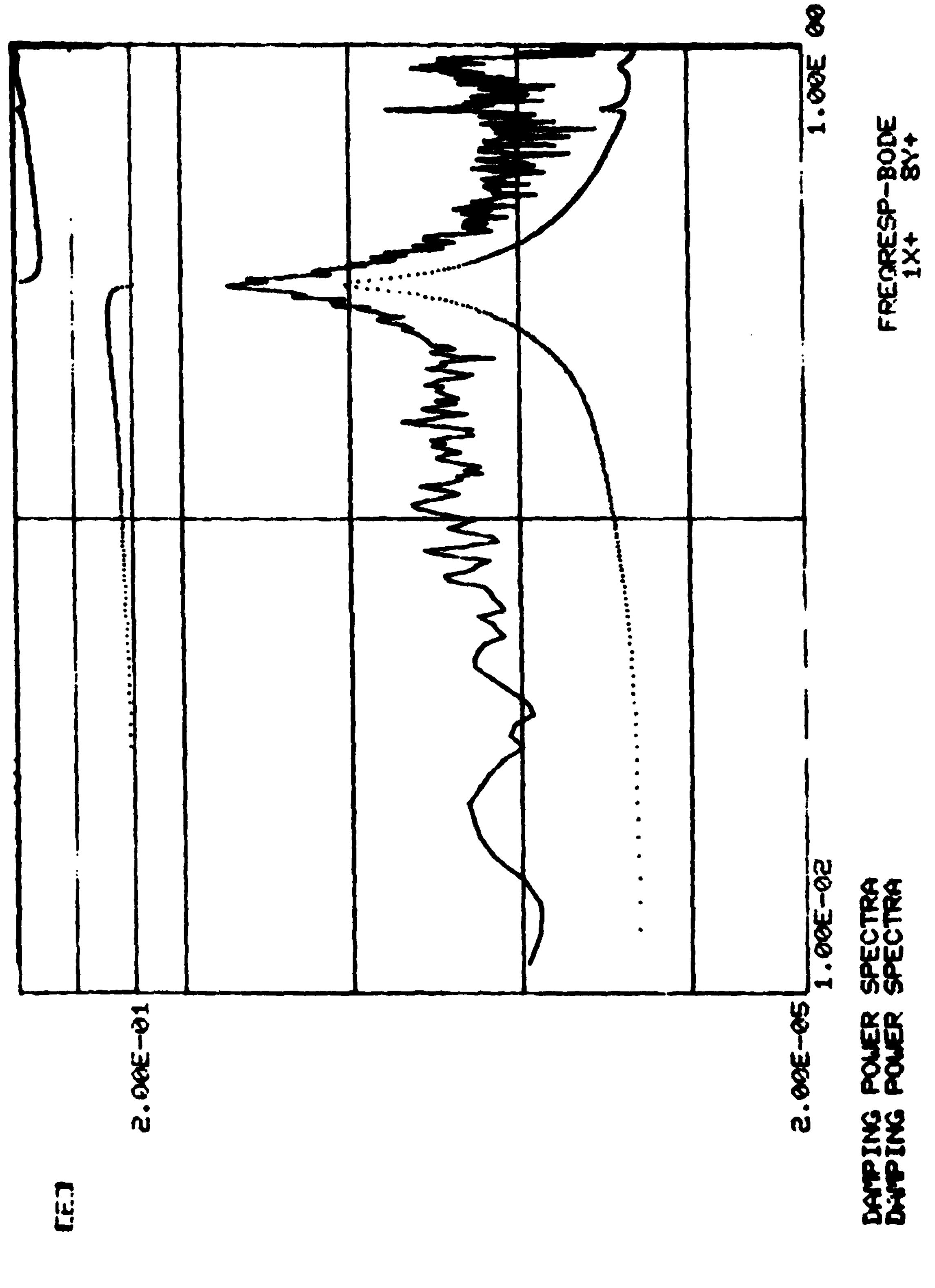
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 7Y+

CORT

ESTIMATED ROOTS (1X+ 8Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.2177	1.000	0.7234E-05	3.142
2	0.1569	1.000	0.3797E-12	1.490
3	0.1278	0.1295	0.2997E-07	-2.093
4	0.1935	0.4098	0.4006E-05	-0.8674
5	0.3122	0.2029E-01	0.4964E-04	0.1340E-02
6	0.3917	0.1880	0.2180E-05	2.215
7	0.4988	0.4302E-01	0.3914E-08	2.664
8	0.6046	0.6774E-01	0.1221E-06	0.1740
9	0.7332	0.7815E-02	0.3819E-06	0.4509E-01
10	0.7651	0.2145	0.2014E-11	-2.952
11	0.9097	0.2905E-01	0.8709E-06	-0.2882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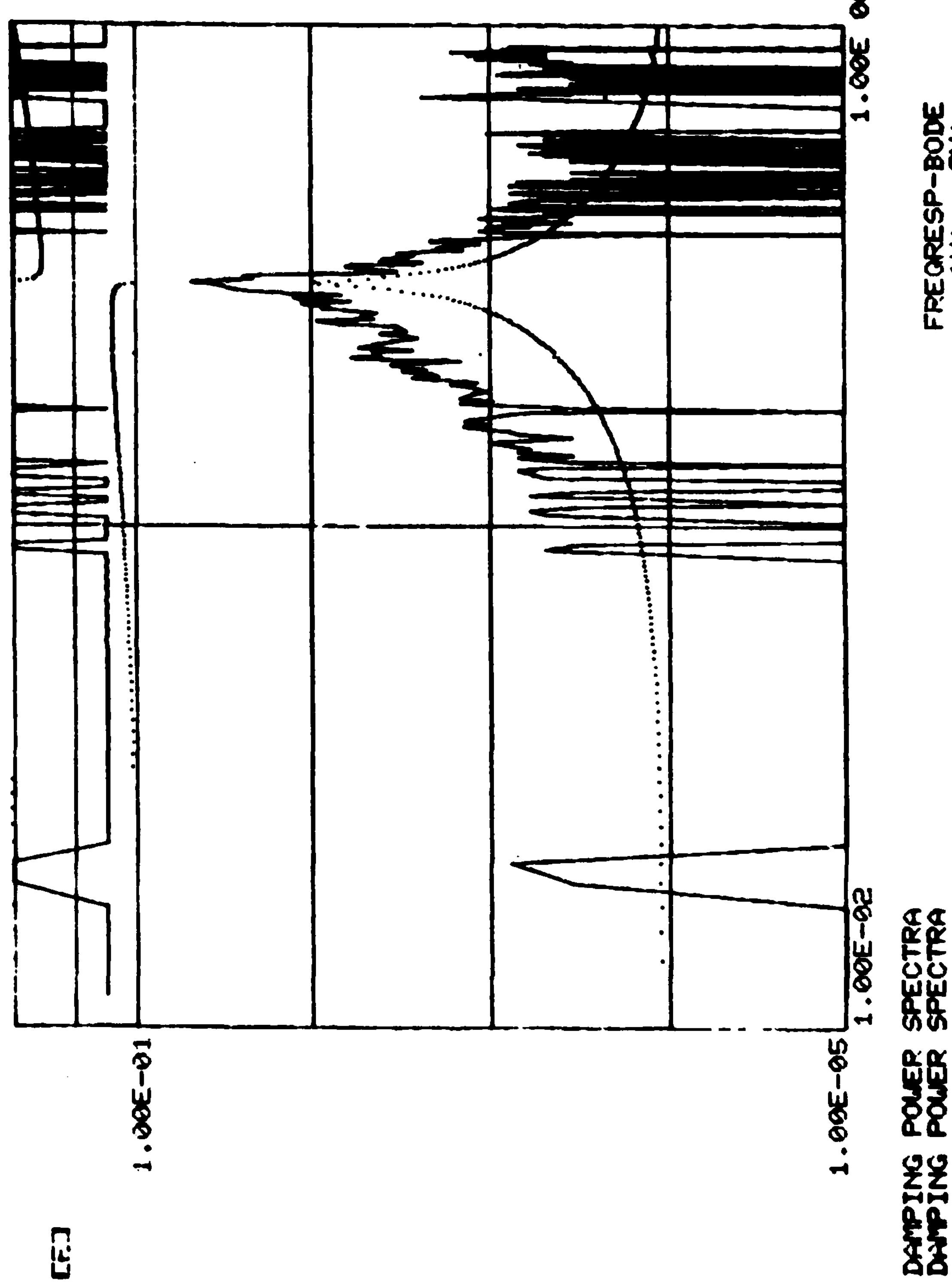
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 8Y+

CORT

ESTIMATED ROOTS (1X+ 9Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.8865E-01	1.000	0.6939E-07	-0.9463E-05
2	0.1675	1.000	0.1584E-11	-2.062
3	0.2207	0.8338E-04	0.2466E-06	-0.2751
4	0.2634	0.1936	0.2852E-05	1.435
5	0.3060	0.1221E-01	0.2735E-04	-0.9509E-01
6	0.3676	0.1213	0.4571E-09	1.401
7	0.5216	0.3723E-01	0.1225E-07	2.847
8	0.6577	0.2817E-01	0.1283E-07	-3.005
9	0.7227	0.3544E-03	0.3357E-07	-0.1519
10	0.8473	0.5341E-01	0.6035E-10	1.307
11	0.8820	0.1095E-01	0.4732E-07	-0.3207



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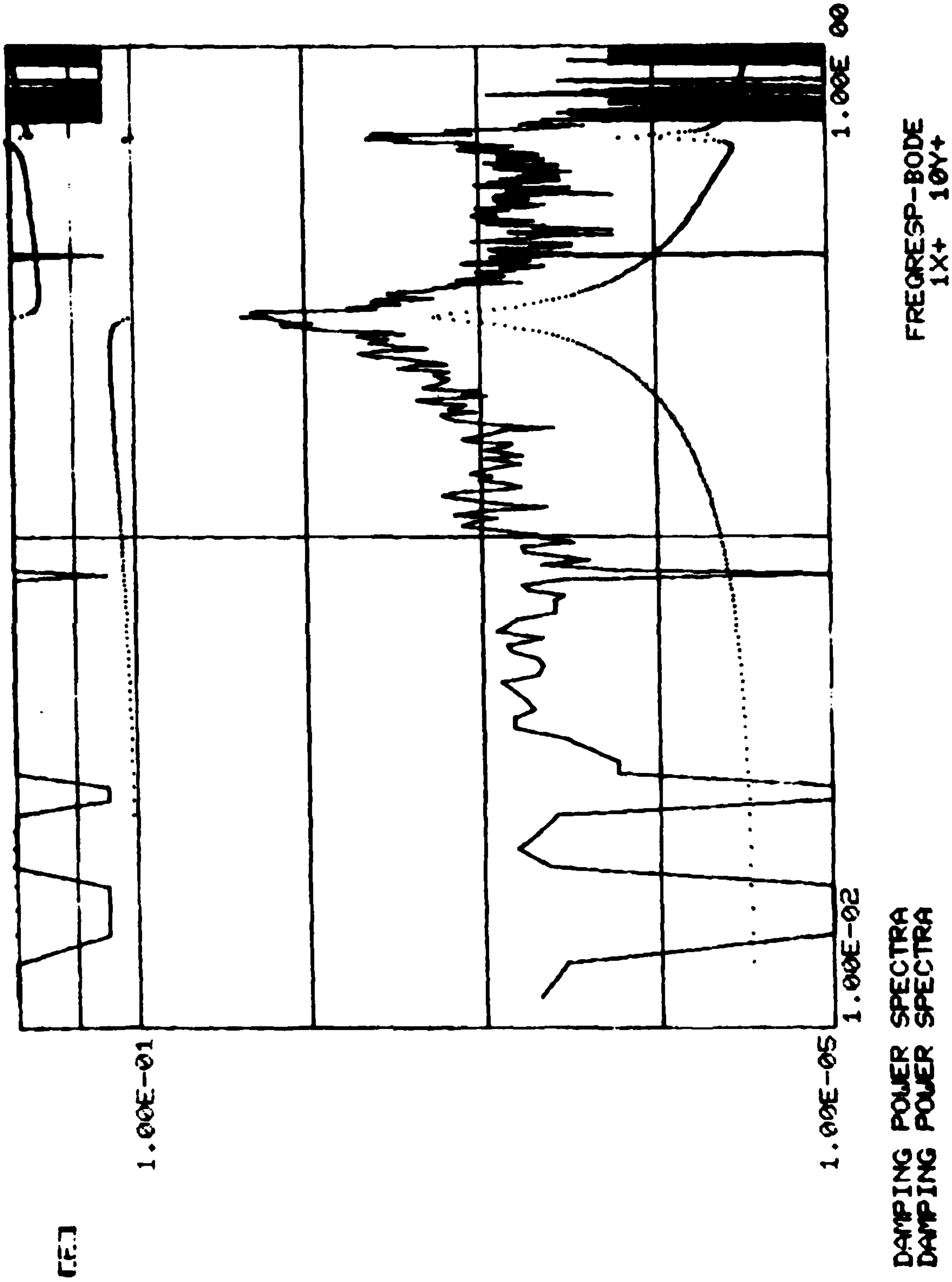
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 9Y+

CORT

ESTIMATED ROOTS (1X+ 10Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1008	1.000	0.4112E-11	-0.9321E-01
2	0.5000E-01	1.000	0.6029E-07	-3.142
3	0.1216	0.2587	0.1463E-08	0.1204E-01
4	0.2736	0.1915	0.8394E-06	1.444
5	0.2815	0.1727E-01	0.6758E-05	-0.1304
6	0.4712	0.1428E-01	0.6853E-08	-1.699
7	0.4743	0.7605E-01	0.5958E-07	0.2749
8	0.6513	0.5144E-02	0.3500E-06	-0.2434
9	0.6600	0.1235E-02	0.6879E-07	0.9560
10	0.7993	0.1070E-02	0.3514E-08	0.1025
11	0.8982	0.2970E-01	0.5115E-10	2.830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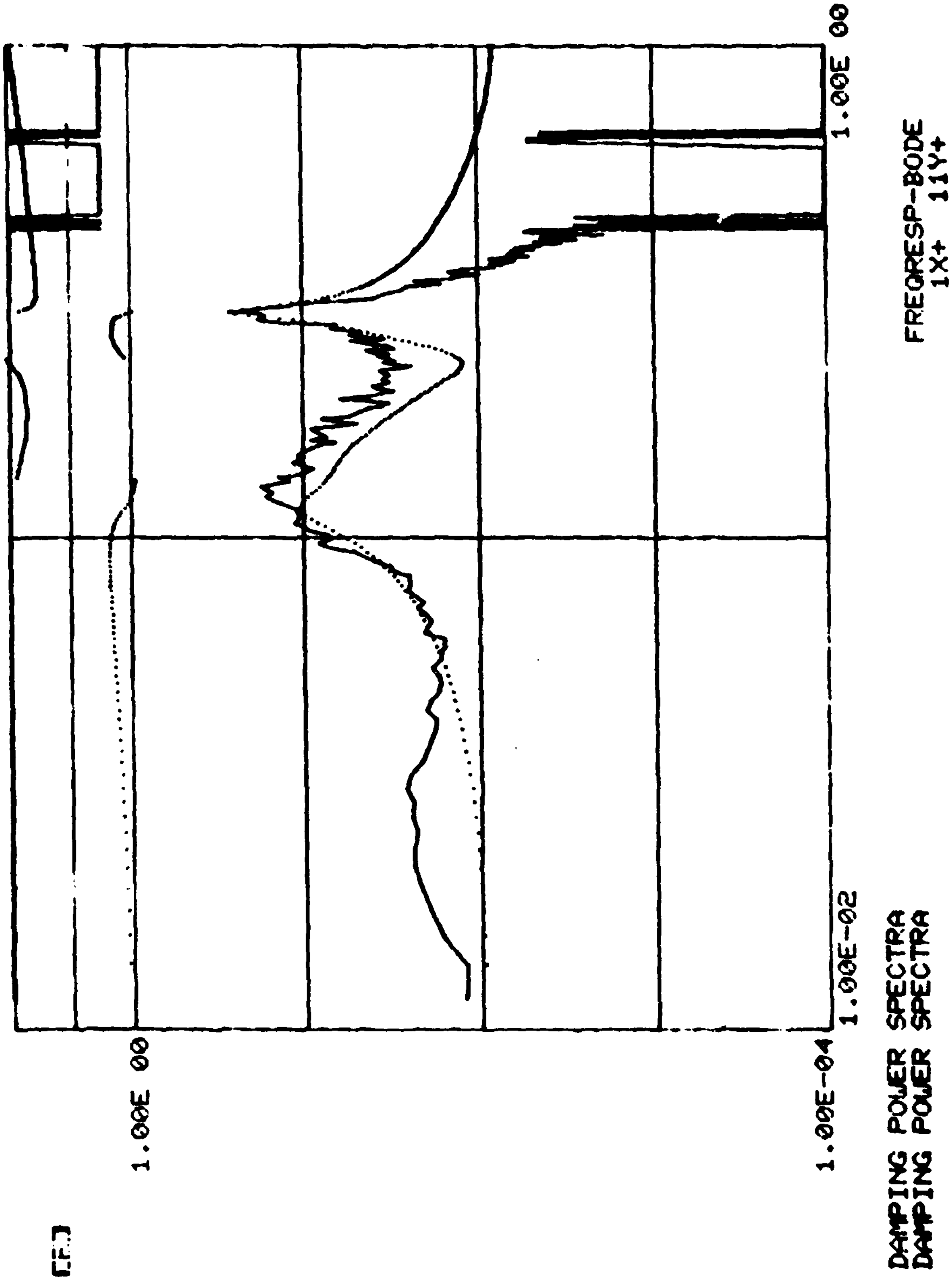
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 10Y+

CORT

ESTIMATED ROOTS (1X+ 11Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.6464E-01	1.000	0.3001E-07	-3.132
2	0.5968	1.000	0.2871E-09	-3.142
3	0.1141	0.9220E-01	0.6186E-03	0.6206
4	0.1468	0.1968	0.6537E-03	-0.5320
5	0.2544	0.9873E-01	0.8956E-04	2.329
6	0.2873	0.1800E-01	0.9974E-03	-0.1200
7	0.4357	0.1221	0.9075E-06	-2.012
8	0.6000	0.5902E-01	0.6185E-08	-2.969
9	0.6526	0.1095E-01	0.5931E-06	-0.9387E-01
10	0.7279	0.3590E-02	0.1112E-07	1.613
11	0.9134	0.2307E-01	0.7845E-09	-1.790



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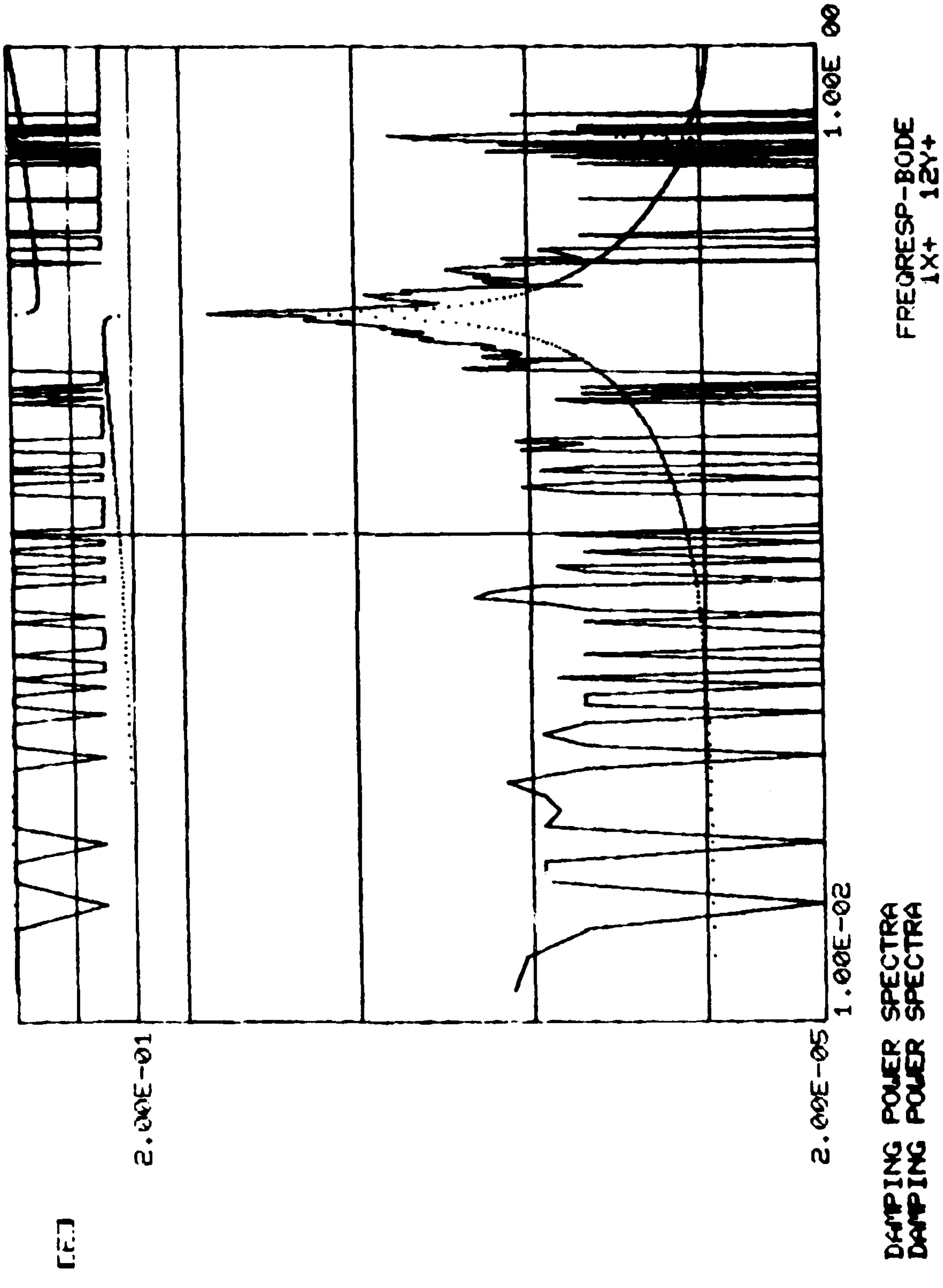
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 11Y+

CORT

ESTIMATED ROOTS (1X+ 12Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1021	0.2276	0.2066E-08	-2.015
2	0.1346	0.5603	0.3331E-07	-1.207
3	0.2289	0.1864	0.3522E-06	-3.127
4	0.2326	0.7188E-02	0.2206E-04	0.2655E-02
5	0.3661	0.9297E-01	0.1589E-06	-2.340
6	0.4430	0.7148E-01	0.1529E-09	-2.003
7	0.6033	0.6499E-01	0.3313E-07	1.700
8	0.6524	0.3664E-02	0.3536E-06	-0.8669E-01
9	0.7259	0.1781E-03	0.3986E-08	0.1702
10	0.8941	0.2437E-01	0.6390E-12	0.3196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FREQRESP-BODE
1X+ 12Y+
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DAMPING POWER SPECTRA

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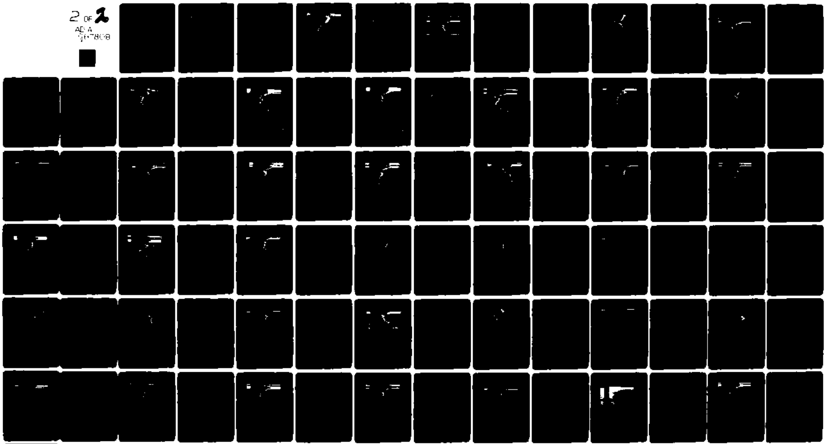

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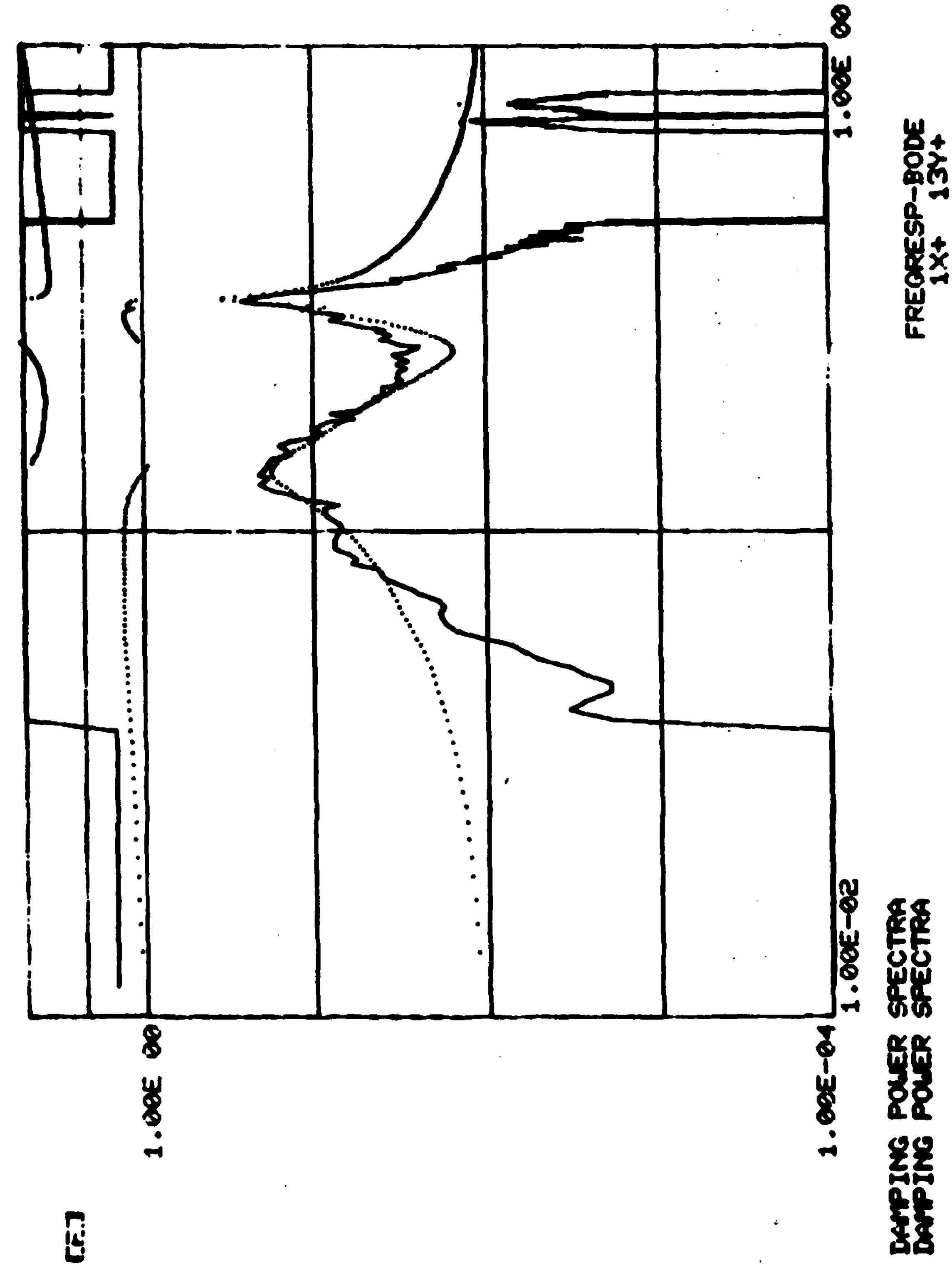
ESTIMATED ROOTS (1X+ 13Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.8317	1.000	0.2356E-04	-3.142
2	0.6624E-01	0.1294	0.9684E-05	1.948
3	0.1328	0.1097	0.1861E-02	0.1360
4	0.1913	0.2380	0.3221E-03	-2.850
5	0.2860	0.6963E-02	0.8389E-04	-0.2571
6	0.3013	0.1470E-01	0.1102E-02	-0.6478E-01
7	0.4438	0.4305E-01	0.1813E-06	-3.024
8	0.6955	0.6166E-02	0.1761E-05	0.1468
9	0.7596	0.6860E-03	0.8202E-06	-0.2189
10	0.7810	0.3946E-02	0.4854E-06	-0.1711E-01
11	1.022	0.2074	0.2773E-06	-0.7022E-03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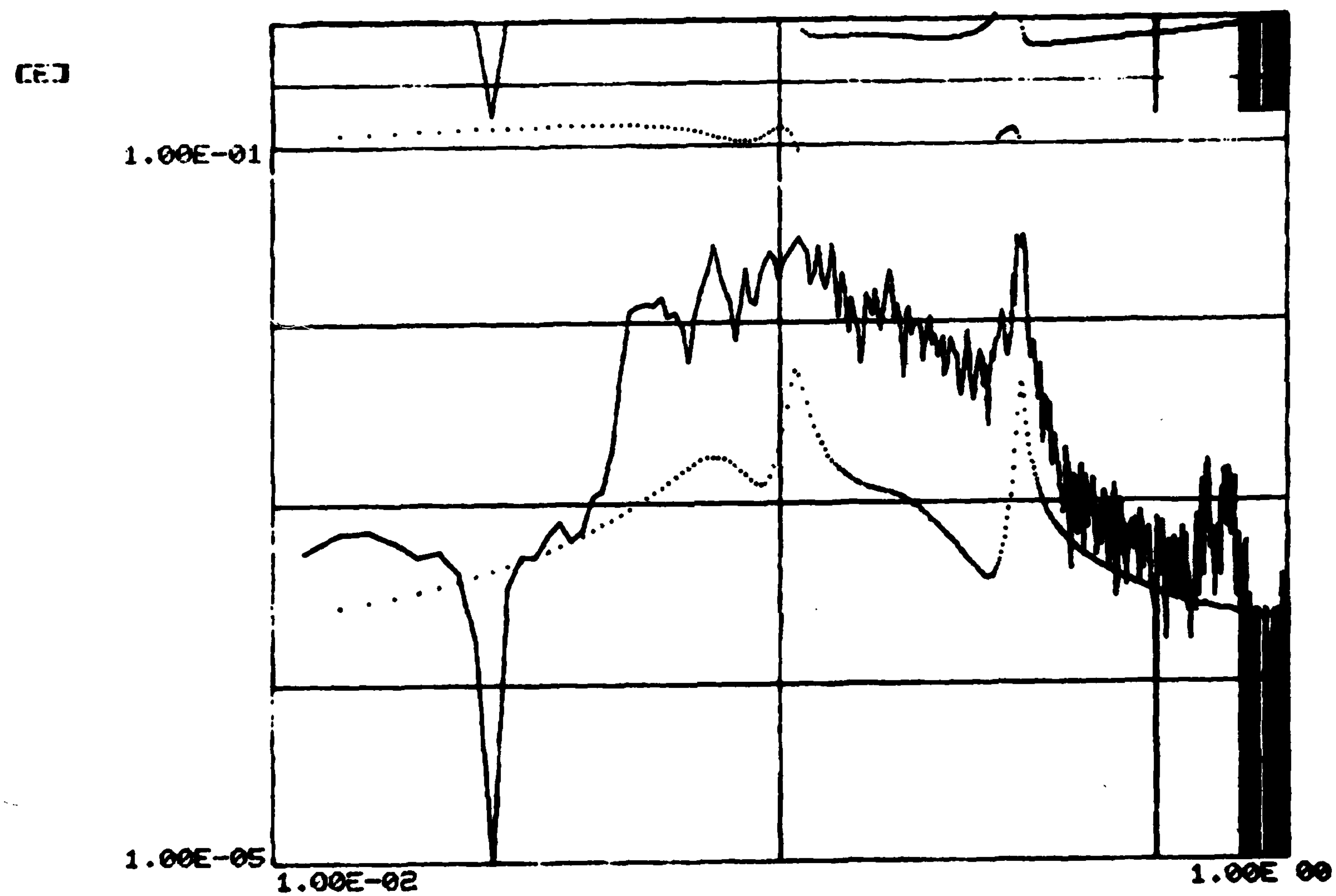
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 13Y+

CORT

ESTIMATED ROOTS (1X+ 15Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.5379	1.000	0.7797E-06	3.135
2	0.1517	1.000	0.1247E-08	1.697
3	0.1264	0.9427E-01	0.1696E-05	0.7650
4	0.2062	0.9048E-01	0.1212E-04	0.2033
5	0.2592	0.1288	0.2455E-04	0.5332
6	0.3189	0.6112E-02	0.1751E-03	-0.7785E-01
7	0.3405	0.3494E-01	0.6981E-05	-0.3087
8	0.4712	0.2000E-01	0.7132E-08	-1.302
9	0.7372	0.2099E-01	0.3682E-08	-1.682
10	0.7443	0.7445E-03	0.1498E-06	0.2653E-01
11	1.025	0.2216	0.2077E-07	-0.9060E-02
12	1.000	0.2202E-02	0.1213E-09	3.114



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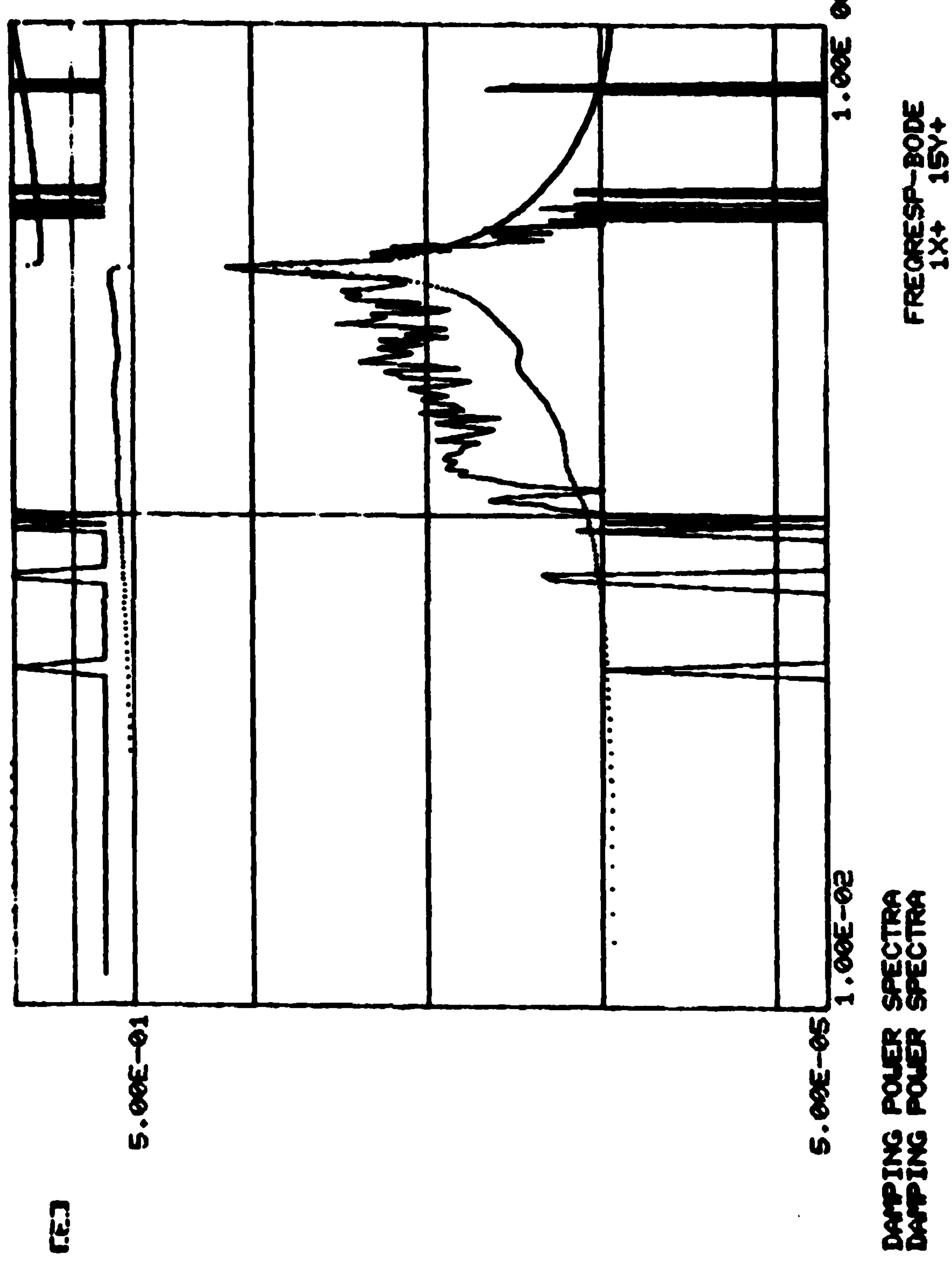
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 14Y+

CORT

ESTIMATED ROOTS (1X+ 14Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.7738E-01	0.2040	0.1751E-04	0.2366
2	0.1075	0.3643E-01	0.1423E-04	0.3452
3	0.1720	0.2191	0.1166E-04	-0.5694
4	0.2993	0.1512E-01	0.1475E-04	-0.9526E-01
5	0.3712	0.1044	0.9396E-07	1.875
6	0.5534	0.2269E-01	0.5126E-08	2.595
7	0.6895	0.7127E-02	0.3882E-07	0.1772
8	0.7659	0.1766E-01	0.8302E-07	-0.1092
9	0.9808	0.5385E-01	0.3917E-11	-1.605
10	1.276	0.6211	0.6250E-06	3.142
11	1.002	0.6570E-01	0.1402E-07	-0.4666E-05



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DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 15Y+

CORT

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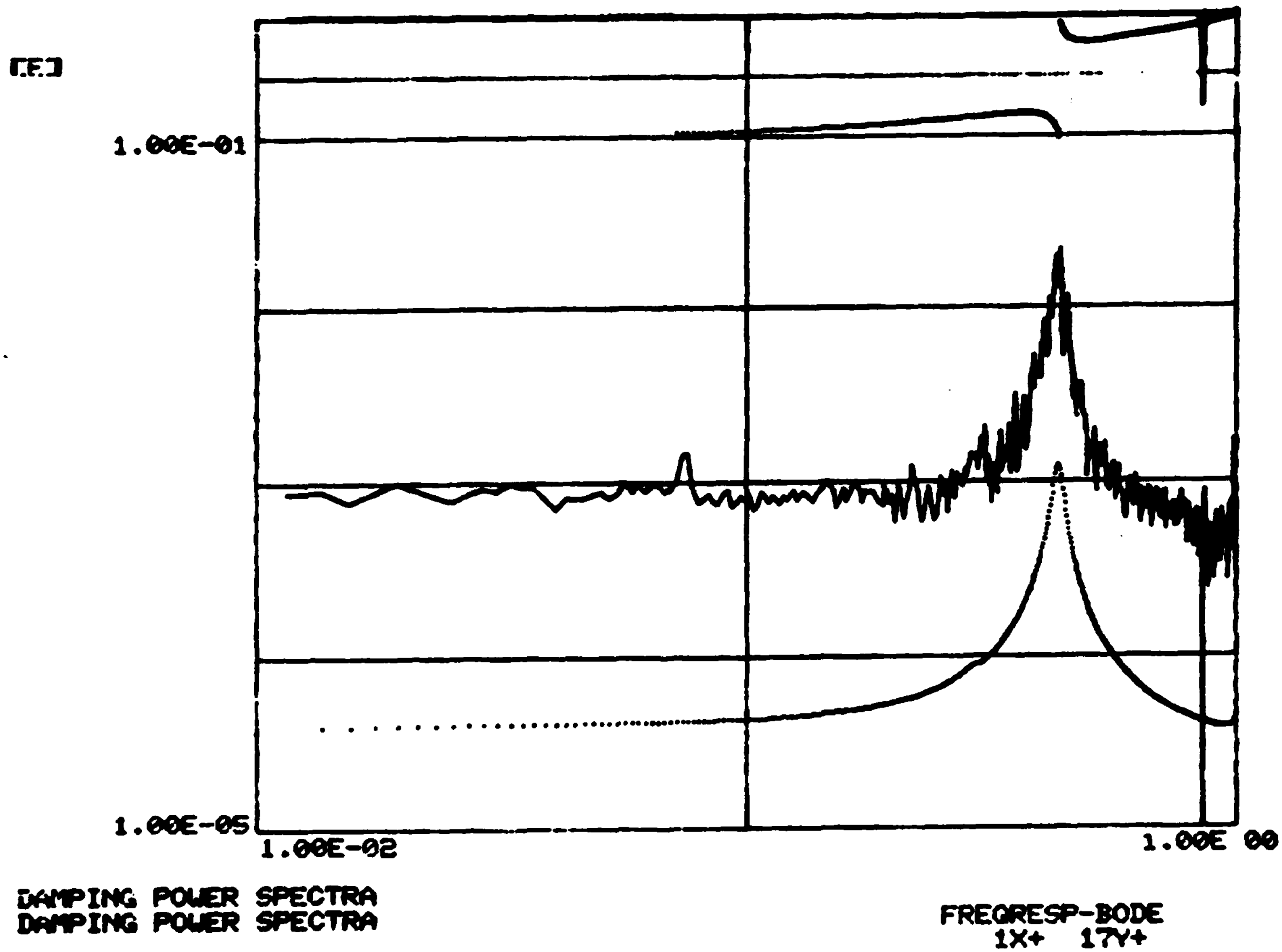
CORT

ESTIMATED ROOTS (1X+ 17Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.4470	1.000	0.1239E-05	0.3283E-06
2	0.1121E-01	1.000	0.3133E-07	-3.142
3	0.2001	1.000	0.1578E-11	2.614
4	0.1032	0.7633E-01	0.4552E-08	-2.441
5	0.2906	0.3685E-01	0.4853E-07	0.3513
6	0.4194	0.1516	0.1769E-05	2.458
7	0.4336	0.2468E-01	0.1006E-04	-0.1011
8	0.5374	0.2211E-01	0.7528E-08	0.4788
9	0.6721	0.5708E-01	0.2087E-09	2.525
10	0.8268	0.1515E-01	0.2059E-08	2.465
11	0.9227	0.6476E-01	0.5524E-07	2.692
12	1.000	0.1693E-02	0.3419E-07	-0.7568E-04



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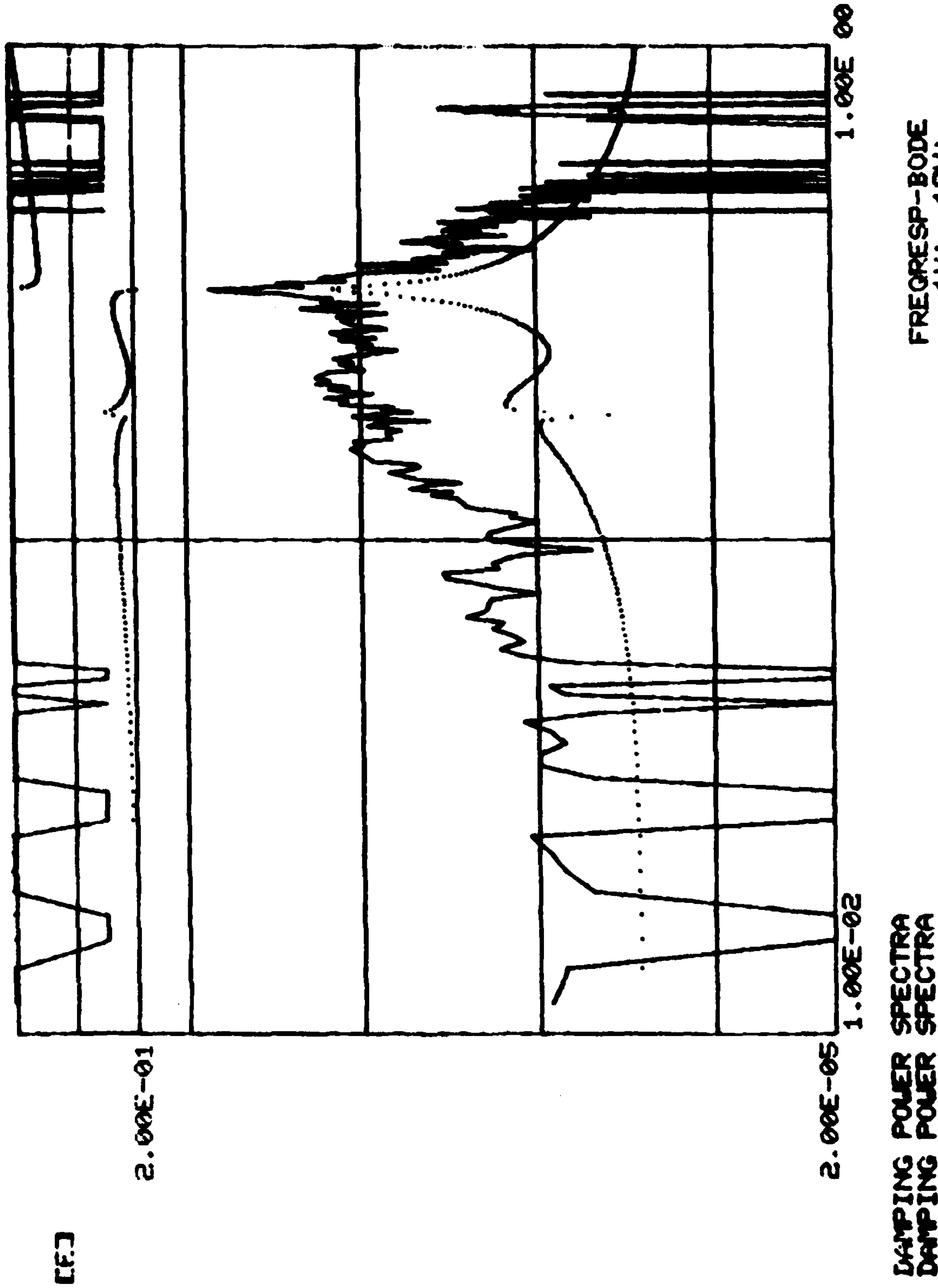
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 17Y+

CORT

ESTIMATED ROOTS (1X+ 18Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1117	0.1740	0.5118E-07	2.353
2	0.1812	0.1696E-01	0.2604E-05	-3.026
3	0.1907	0.1407	0.1988E-04	0.3709
4	0.3222	0.1372E-01	0.4654E-04	-0.1281
5	0.3660	0.1382	0.3807E-08	1.137
6	0.3988	0.1370	0.1982E-05	-0.2413
7	0.5080	0.9209E-01	0.7473E-07	-2.841
8	0.7431	0.4247E-02	0.1463E-06	0.3753E-01
9	0.7681	0.4677E-01	0.2545E-09	-1.087
10	0.8164	0.2827E-01	0.2645E-08	-2.190



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FREQRESP-BODE
1X+ 18Y+
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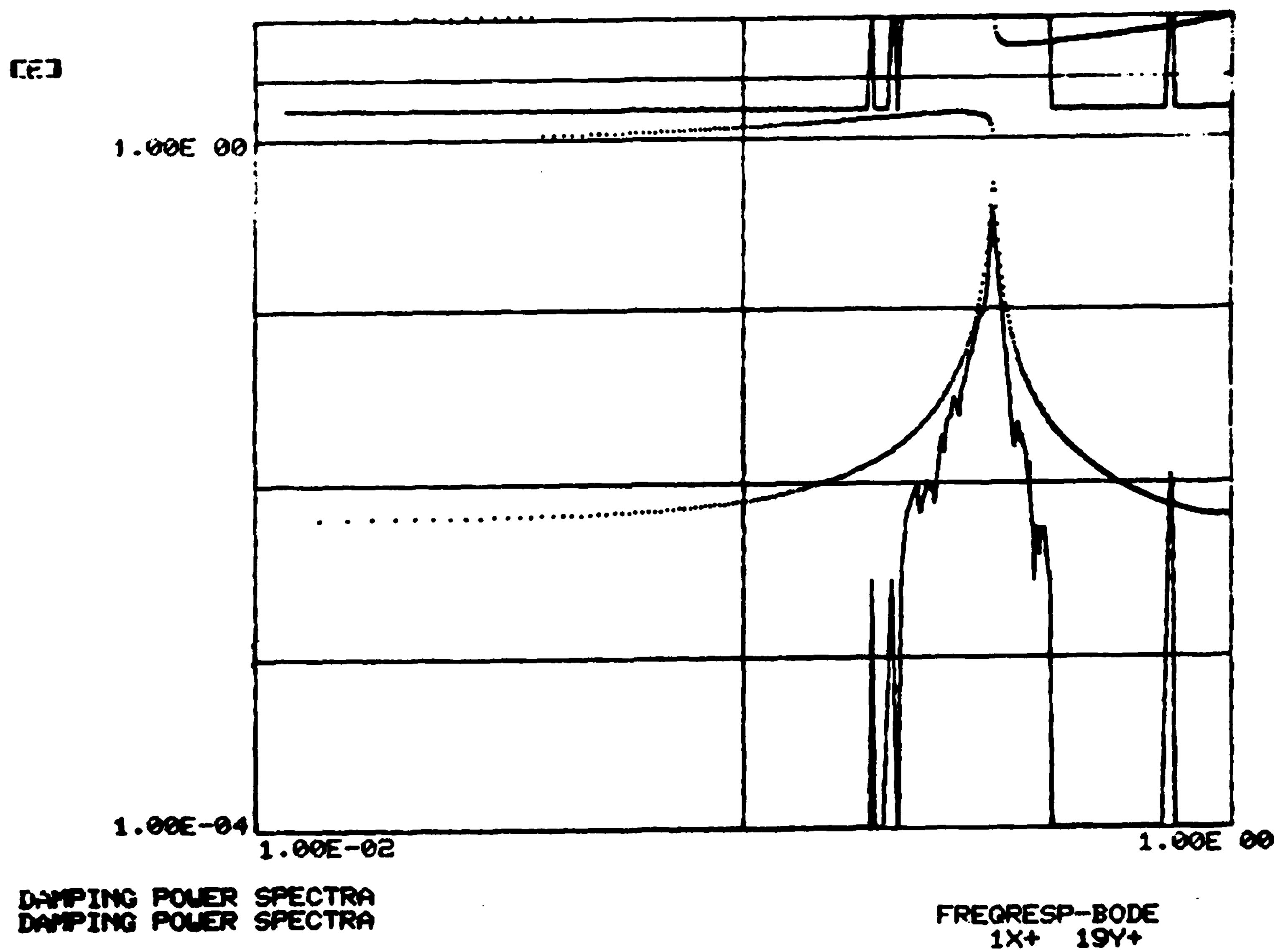
CORT

ESTIMATED ROOTS (1X+ 19Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.5327E-01	1.000	0.9115E-08	-3.103
2	0.2109	0.8537E-01	0.8691E-07	0.8744
3	0.2492	0.2618	0.1181E-04	-2.623
4	0.3066	0.1243	0.2094E-03	1.829
5	0.3262	0.1255E-01	0.1647E-02	-0.1196
6	0.3624	0.2151	0.5032E-08	2.941
7	0.4290	0.8262E-01	0.1059E-05	-3.123
8	0.7451	0.1263E-01	0.3517E-06	2.279
9	0.7527	0.3942E-02	0.1407E-05	-0.1983
10	0.7964	0.6189E-02	0.3169E-08	2.599
11	1.002	0.6466E-01	0.4123E-10	-2.949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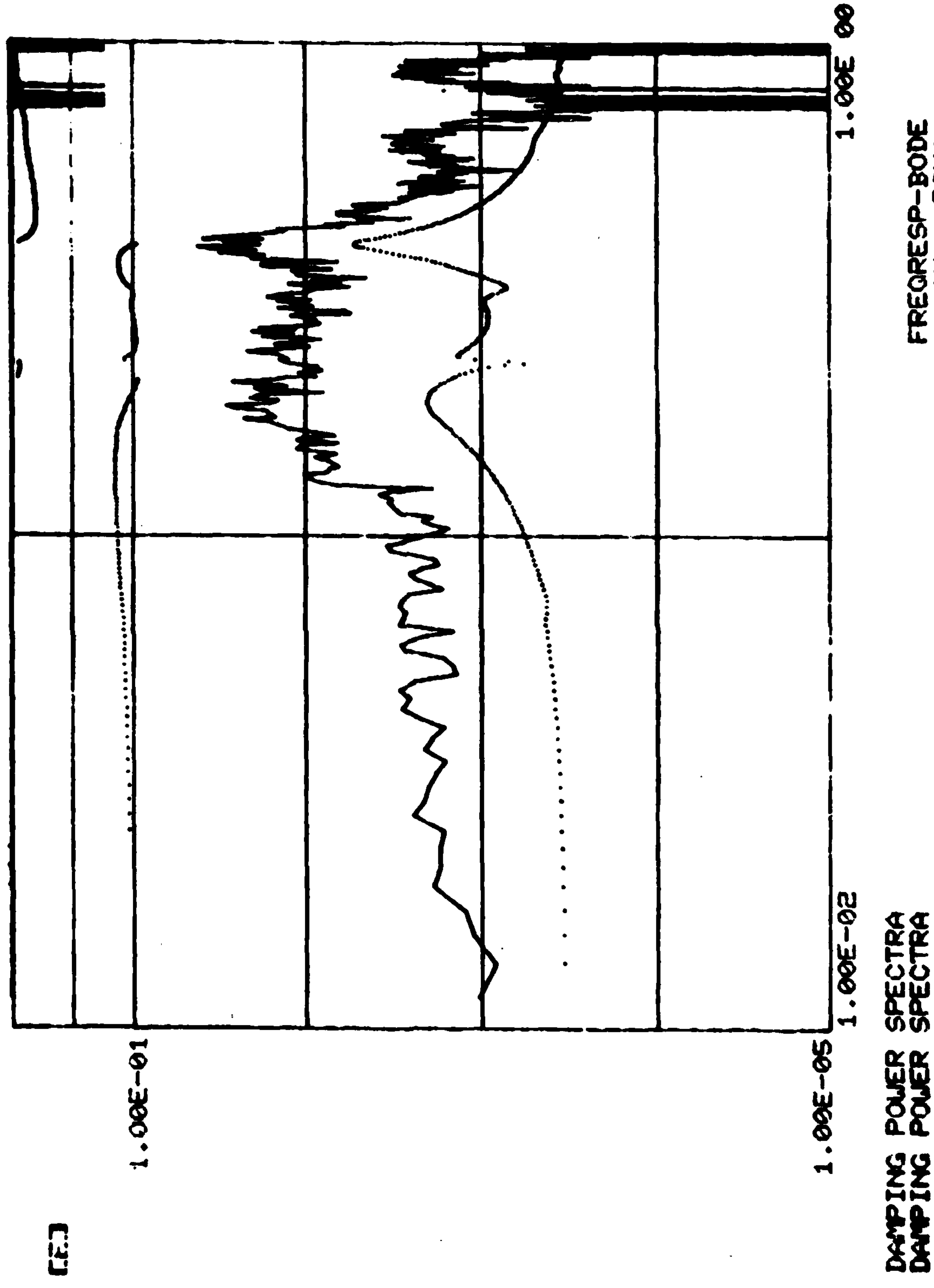
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 19Y+

BLOUGH

ESTIMATED ROOTS (1X+ 20Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.7152E-01	0.8137E-01	0.1511E-06	-1.484
2	0.1893	0.1354	0.3294E-04	0.2602
3	0.2278	0.1751E-01	0.2860E-05	1.936
4	0.3114	0.4606E-01	0.3870E-05	-1.334
5	0.3908	0.3390E-01	0.4802E-04	-0.1409
6	0.4136	0.7364E-01	0.1466E-06	0.3749
7	0.6378	0.5630E-01	0.8730E-06	-0.2040
8	0.7221	0.5732E-01	0.2938E-09	-0.5883
9	0.8652	0.2257E-01	0.4188E-06	-0.4651E-01
10	0.9101	0.9579E-02	0.1289E-06	0.1459



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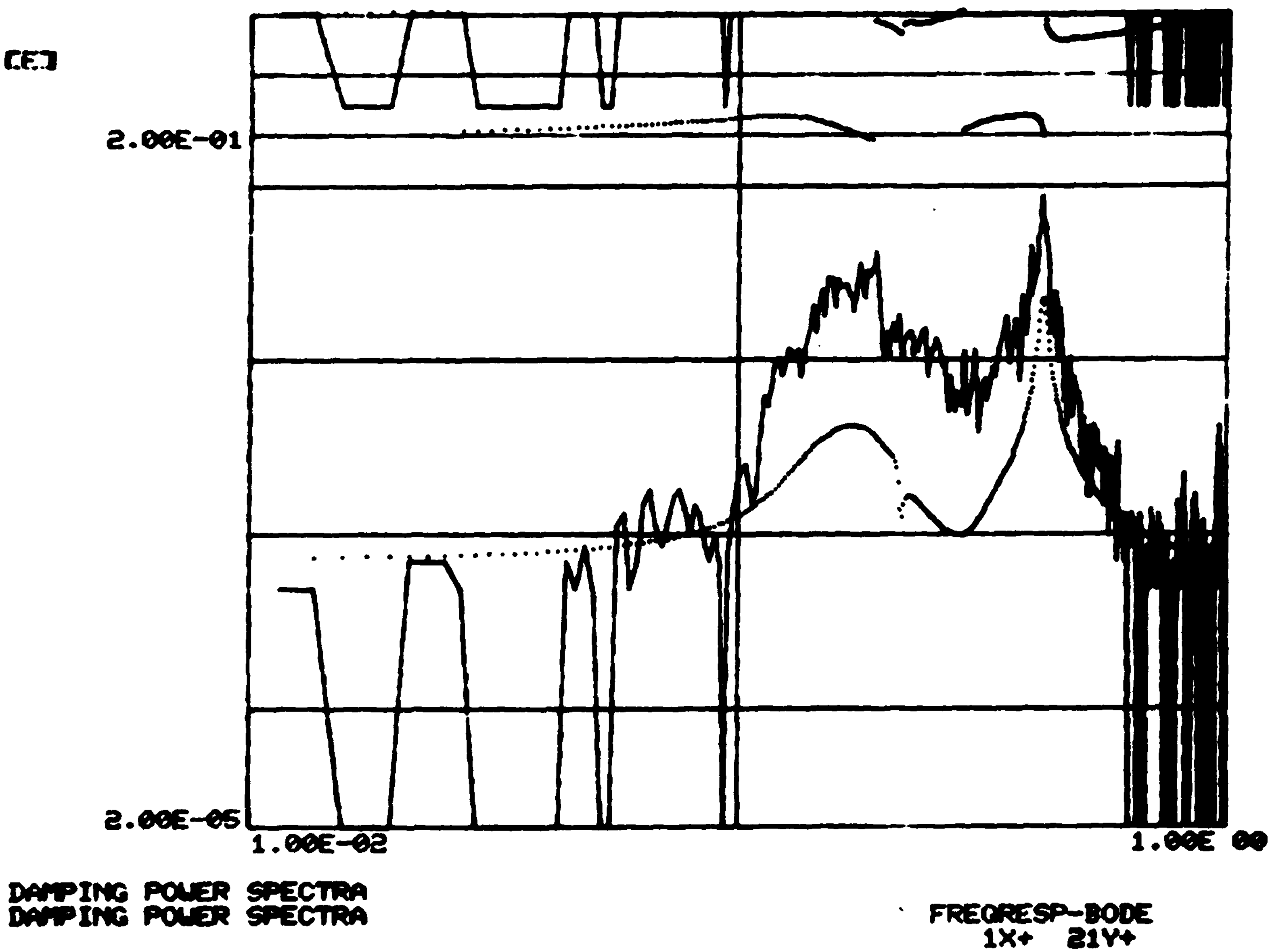
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 20Y+

BLOUGH

ESTIMATED ROOTS (1X+ 21Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1414	0.2049	0.2473E-04	2.100
2	0.1805	0.1796	0.7979E-04	-0.1956
3	0.2145	0.1727E-01	0.3261E-05	-3.129
4	0.3352	0.1418	0.1136E-04	2.018
5	0.4206	0.1742E-01	0.1202E-03	-0.1016
6	0.4831	0.1372E-01	0.3540E-06	-0.1437
7	0.6083	0.1091	0.7492E-06	-2.030
8	0.7550	0.8718E-03	0.2490E-07	-2.394
9	0.9528	0.3461E-02	0.1192E-06	0.7982
10	1.013	0.1610	0.6427E-11	-0.5952E-03
11	1.235	0.5865	0.1125E-10	-0.7184E-08



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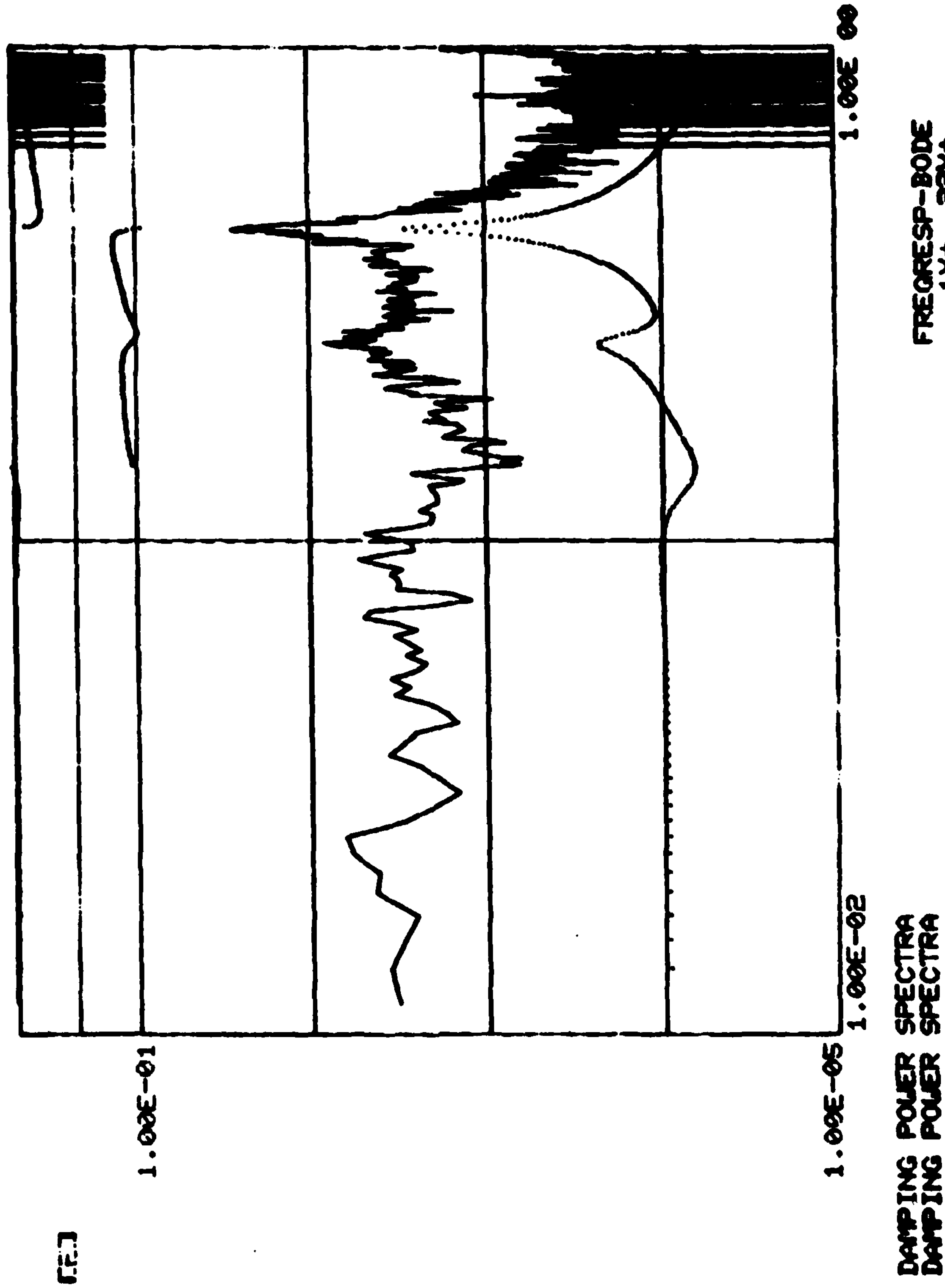
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 21Y+

BLOUGH

ESTIMATED ROOTS (1X+ 22Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1093	0.8826	0.8635E-10	1.454
2	0.1253	0.2459	0.1047E-05	-1.634
3	0.2526	0.4639E-01	0.1166E-05	-0.2535E-01
4	0.2901	0.4253	0.2328E-05	-0.7444E-03
5	0.4290	0.1170E-01	0.1095E-04	-0.5869E-01
6	0.5123	0.8926E-01	0.6327E-07	2.705
7	0.6685	0.2616E-01	0.5804E-09	0.3091E-02
8	0.7399	0.4230	0.2855E-12	1.636
9	0.7842	0.2938E-01	0.1520E-07	0.4569
10	1.000	0.9546E-02	0.6712E-07	0.2997E-05
11	1.004	0.9196E-01	0.1839E-11	2.758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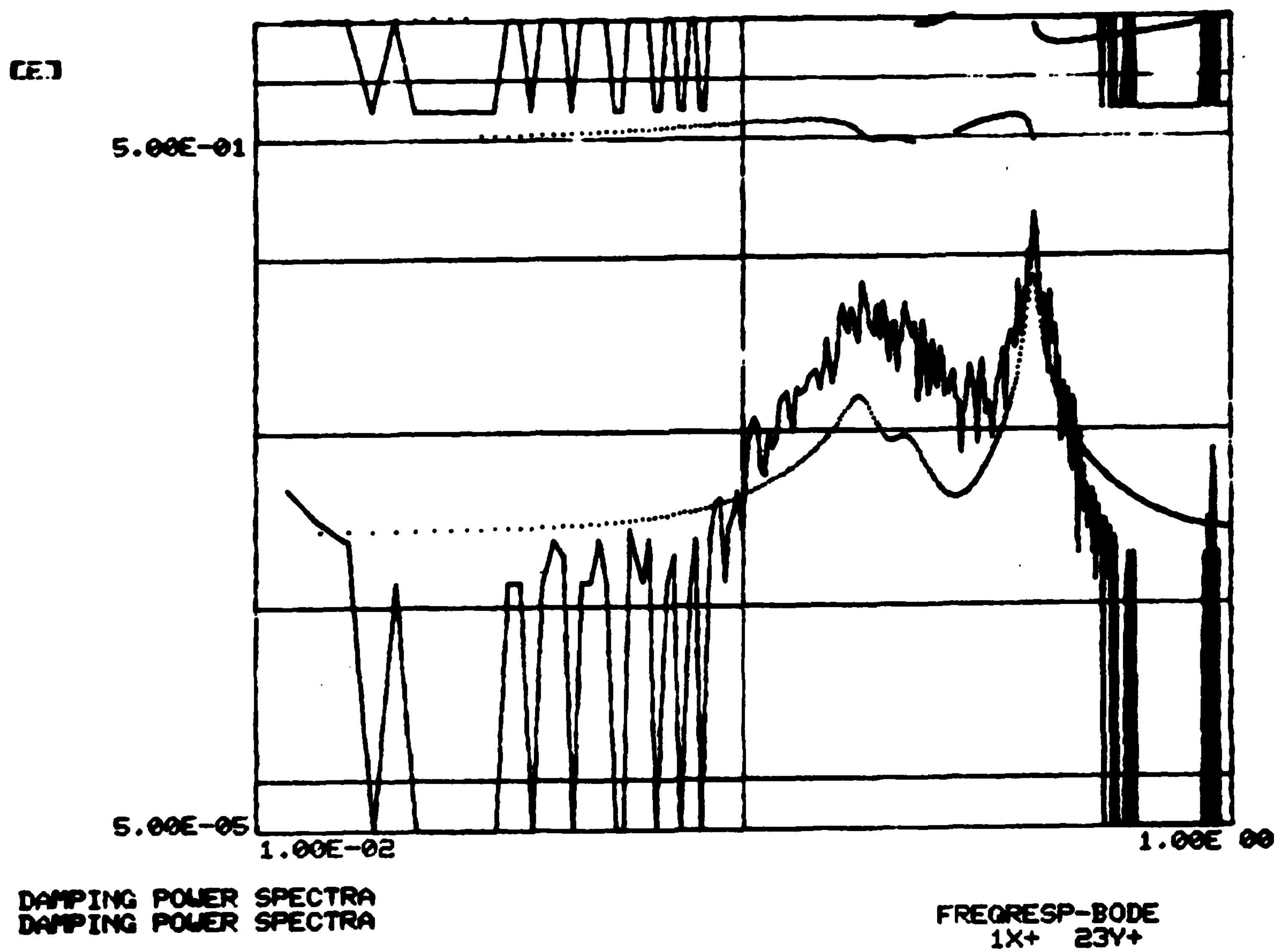
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 22Y+

BLOUGH

ESTIMATED ROOTS (1X+ 23Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1128	0.2908	0.9288E-05	2.345
2	0.1753	0.8071E-01	0.1227E-03	0.7743E-01
3	0.2172	0.9195E-01	0.7378E-04	0.2465E-01
4	0.3690	0.6879E-01	0.1863E-05	-1.893
5	0.3929	0.2325E-01	0.5114E-03	0.8326E-01
6	0.4265	0.1070	0.6964E-04	-2.159
7	0.5679	0.5747E-01	0.4137E-08	-0.3166
8	0.6642	0.1390E-01	0.2653E-08	-2.143
9	0.8948	0.2204E-01	0.9318E-07	1.605
10	0.9210	0.3424E-02	0.3168E-06	-0.1865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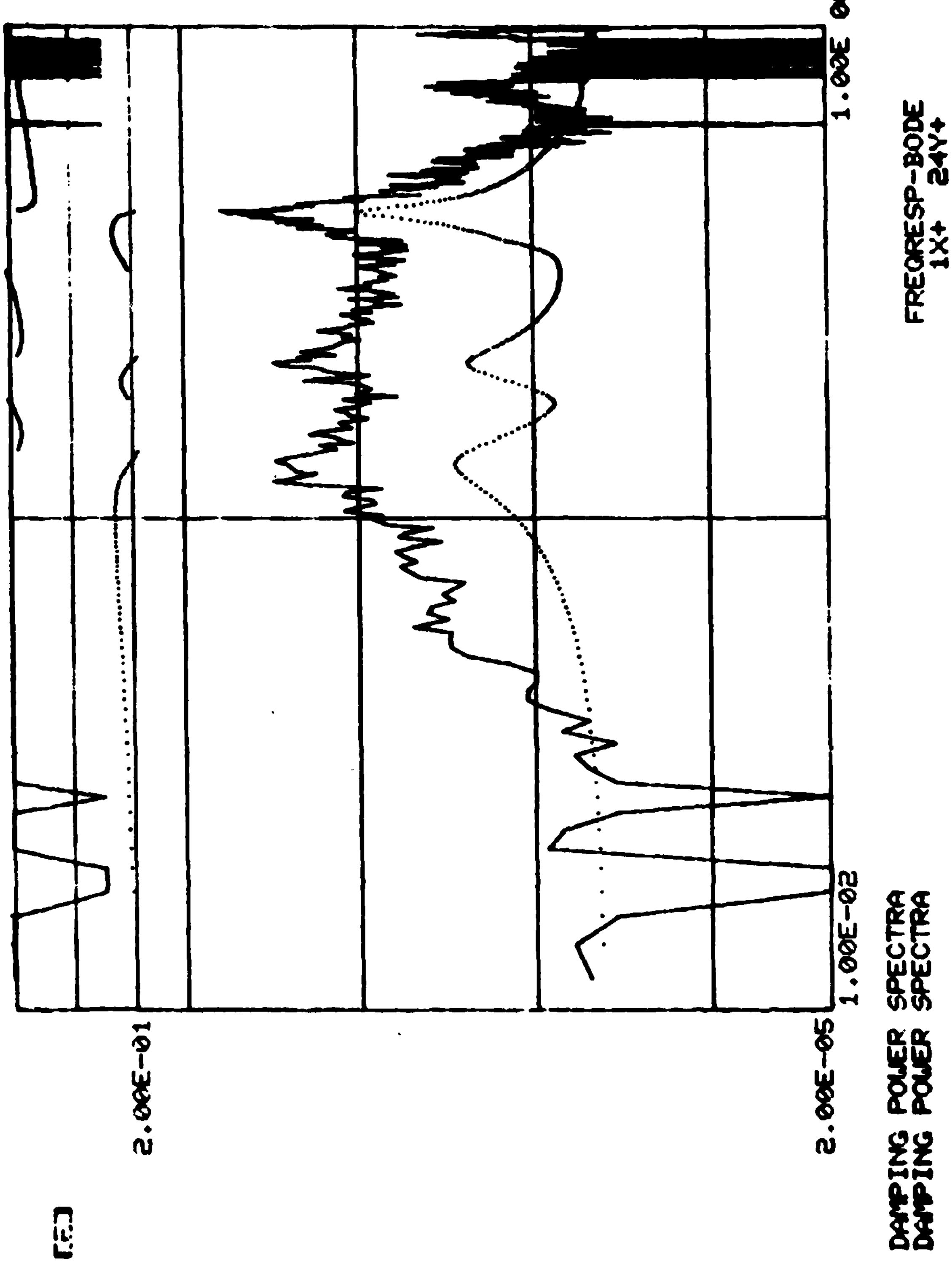
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 23Y+

BLOUGH

ESTIMATED ROOTS (1X+ 24Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1247	0.2030	0.1263E-06	-0.6582
2	0.1305	0.1012	0.2375E-04	0.8417E-01
3	0.2049	0.6762E-01	0.1918E-04	0.3804
4	0.3575	0.7346E-01	0.5326E-05	-3.053
5	0.4204	0.1842E-01	0.5710E-04	-0.1955
6	0.4470	0.1451	0.8044E-05	0.3896
7	0.5632	0.3497E-01	0.7127E-07	-1.630
8	0.7738	0.2319E-01	0.7506E-06	-1.576
9	0.7767	0.4760E-01	0.1047E-05	0.8562
10	0.9696	0.6407E-02	0.2747E-06	-0.7438E-01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1.00E-02
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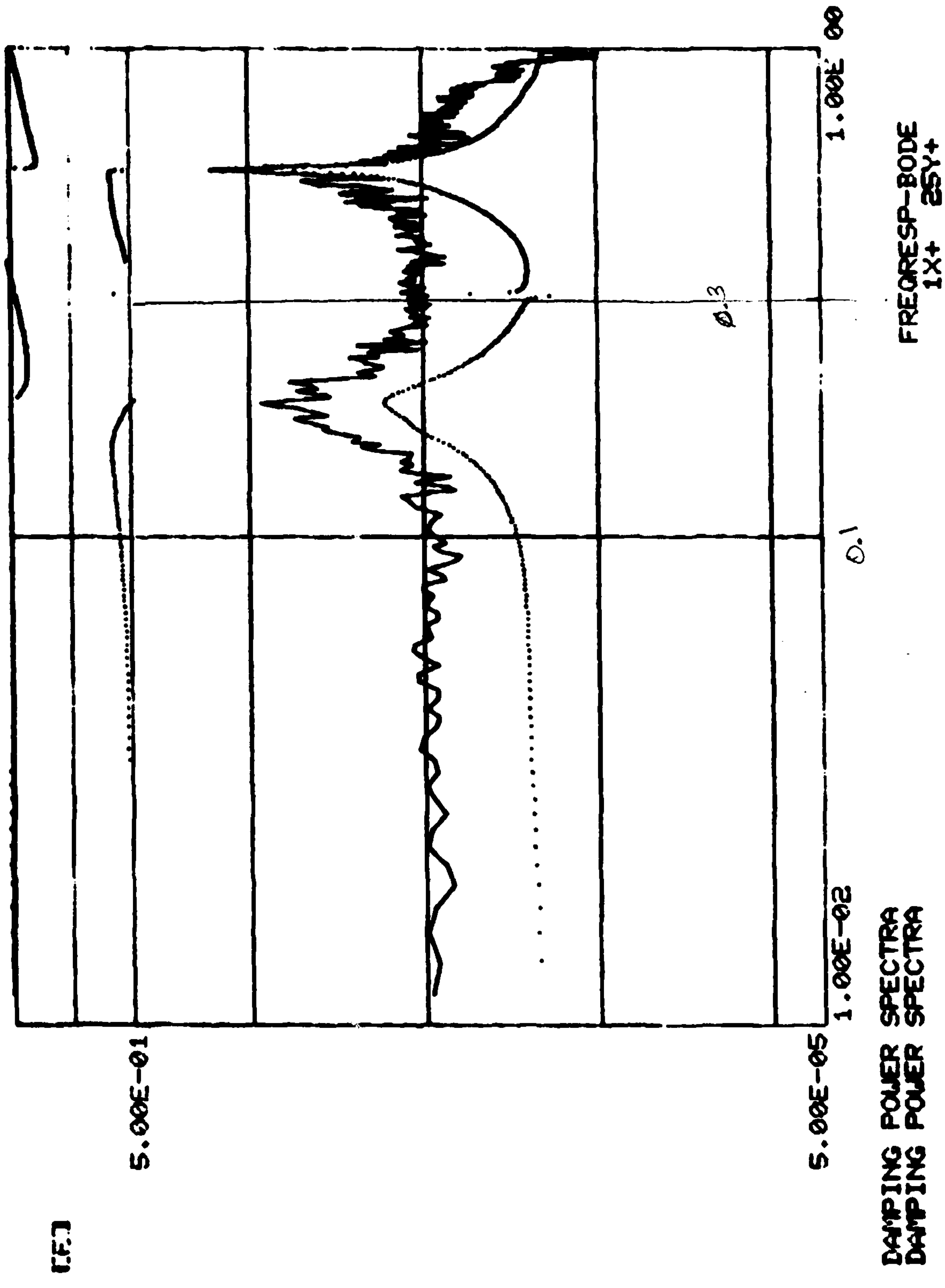
FREQRESP-BODE
1X+ 24Y+
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

RYERSON

ESTIMATED ROOTS (1X+ 25Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.3133E-01	1.000	0.2305E-04	-3.142
2	0.7040E-01	1.000	0.5671E-04	-0.6415E-05
3	0.1676	0.6174E-01	0.2496E-04	0.7649
4	0.1894	0.6891E-01	0.1324E-03	-0.2733E-02
5	0.3144	0.8692E-03	0.1613E-05	1.428
6	0.4616	0.1291	0.6232E-08	1.771
7	0.5443	0.6958E-01	0.8188E-04	0.1904
8	0.5659	0.4828E-02	0.2379E-03	-0.1154
9	0.7142	0.5182E-01	0.5104E-05	-0.4971E-02
10	0.8373	0.4858E-01	0.3374E-05	-0.8606
11	0.9236	0.1538E-01	0.4739E-06	-0.3539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1.00E-02
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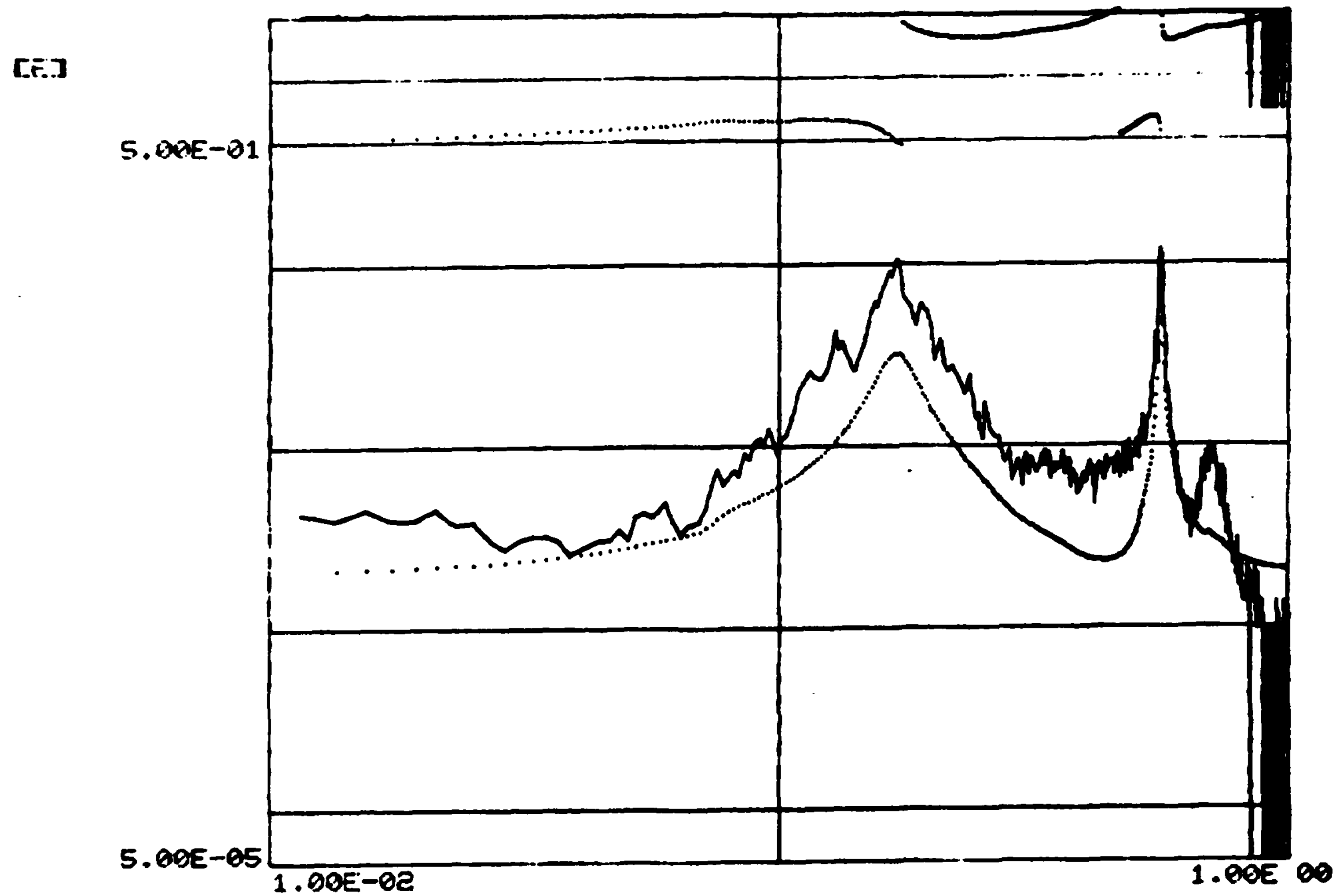
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 25Y+

RYERSON

ESTIMATED ROOTS (1X+ 26Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.7534E-01	0.1025	0.2949E-05	2.860
2	0.1435	0.2309	0.4129E-06	-2.907
3	0.1701	0.9171E-01	0.3654E-03	0.6384E-01
4	0.2887	0.2019	0.2992E-04	2.586
5	0.3834	0.1355	0.8216E-05	-2.787
6	0.5293	0.1454	0.1080E-04	-2.172
7	0.5622	0.5392E-02	0.1745E-03	-0.2730E-01
8	0.6995	0.1700E-01	0.2782E-05	-0.2744
9	0.7298	0.3203E-01	0.4031E-05	-0.1079
10	0.8887	0.1884E-01	0.1480E-06	-1.764



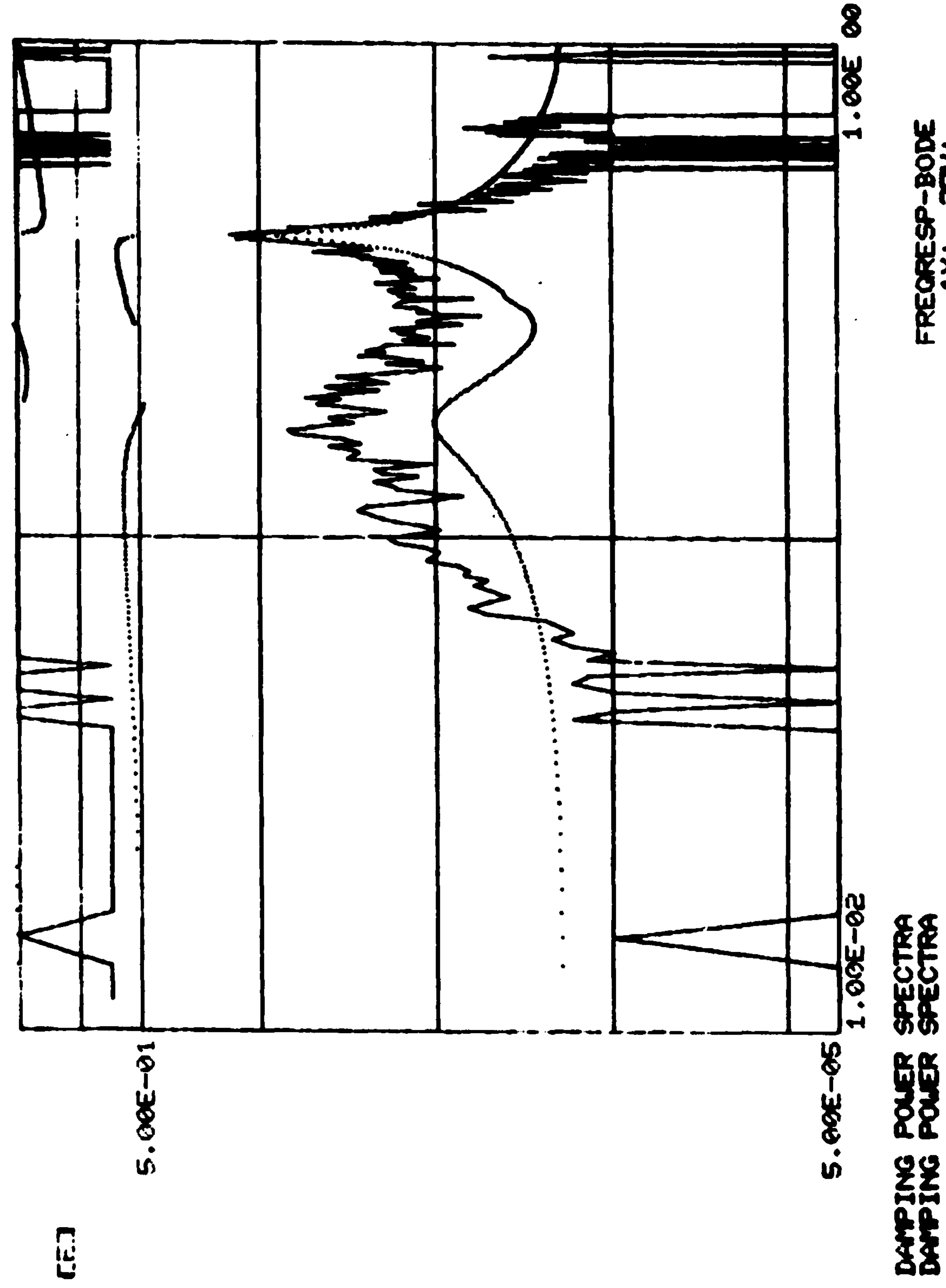
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 26Y+

BLOUGH

ESTIMATED ROOTS (1X+ 27Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.2886	1.000	0.5384E-04	3.142
2	0.3786E-01	1.000	0.6927E-07	-0.4981E-02
3	0.1066	0.3885	0.1331E-06	-0.2127
4	0.1736	0.1469	0.1613E-03	0.8456E-01
5	0.3023	0.3589E-01	0.3115E-05	1.189
6	0.4070	0.1432E-01	0.3519E-03	-0.4453E-01
7	0.4732	0.1618	0.1192E-04	2.320
8	0.6641	0.8577E-01	0.2570E-05	2.836
9	0.6793	0.3569E-01	0.2113E-05	-0.2599
10	0.7284	0.1387	0.5040E-10	2.015
11	0.9426	0.3286E-02	0.2343E-06	0.5625E-02



5.00E-01
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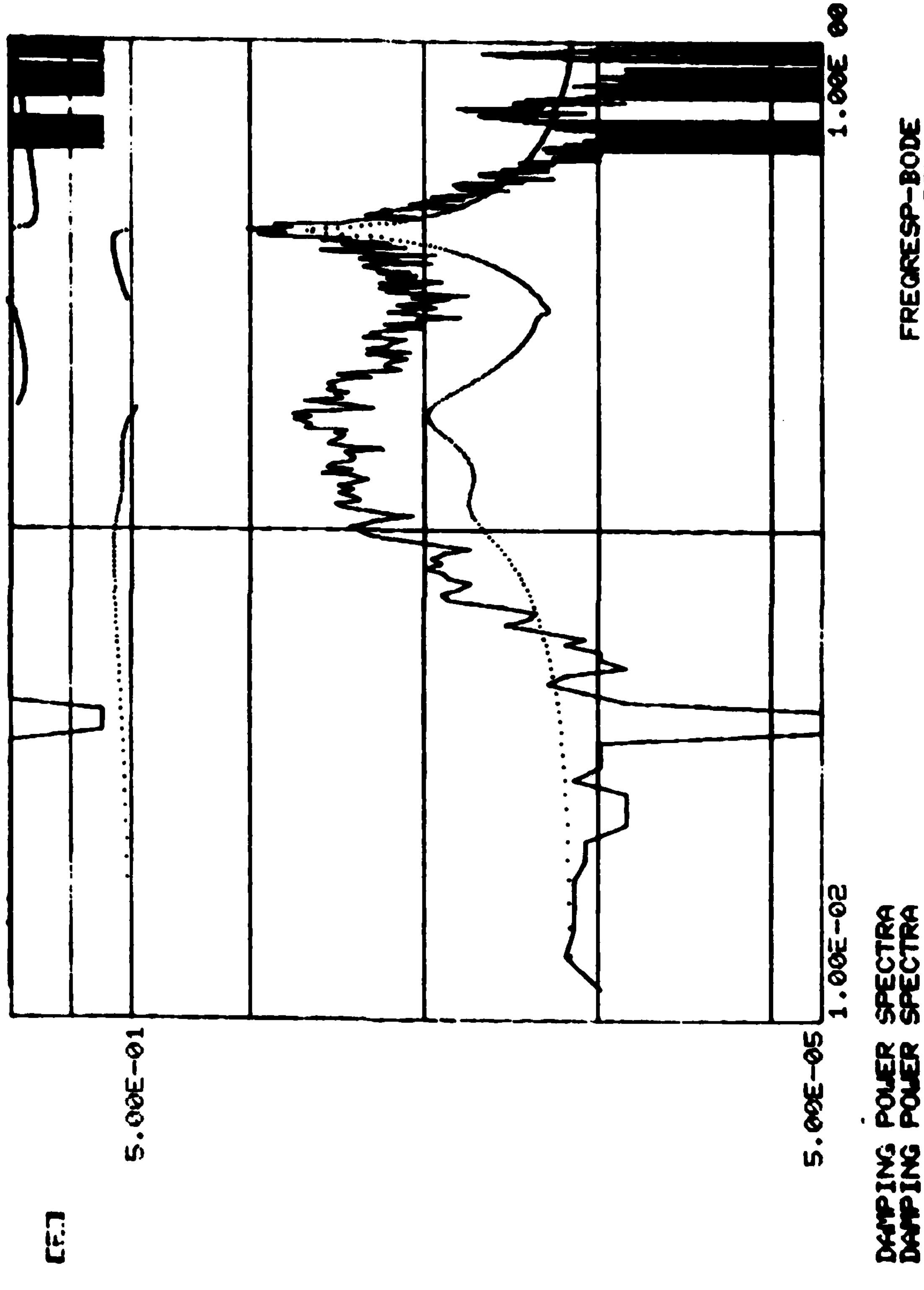
1.00E-02
1.00E 00
FREQRESP-BODE
1X+ 27Y+
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

BLOUGH

ESTIMATED ROOTS (1X+ 28Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.7227E-01	1.000	0.3216E-05	-0.4574E-03
2	0.1120	0.1571	0.3406E-04	0.7363
3	0.1734	0.1109	0.1099E-03	-0.2755E-01
4	0.2791	0.2330E-01	0.1504E-05	-2.080
5	0.4015	0.5105E-01	0.5662E-04	-0.5528
6	0.4078	0.1202E-01	0.1623E-03	0.7603E-01
7	0.5518	0.6453E-01	0.4345E-08	-0.4238
8	0.7307	0.4777E-01	0.4136E-05	-0.5012
9	0.7339	0.7068E-01	0.3732E-05	2.372
10	0.9412	0.1906E-02	0.2411E-06	0.1268
11	1.000	0.1585E-01	0.8715E-08	3.141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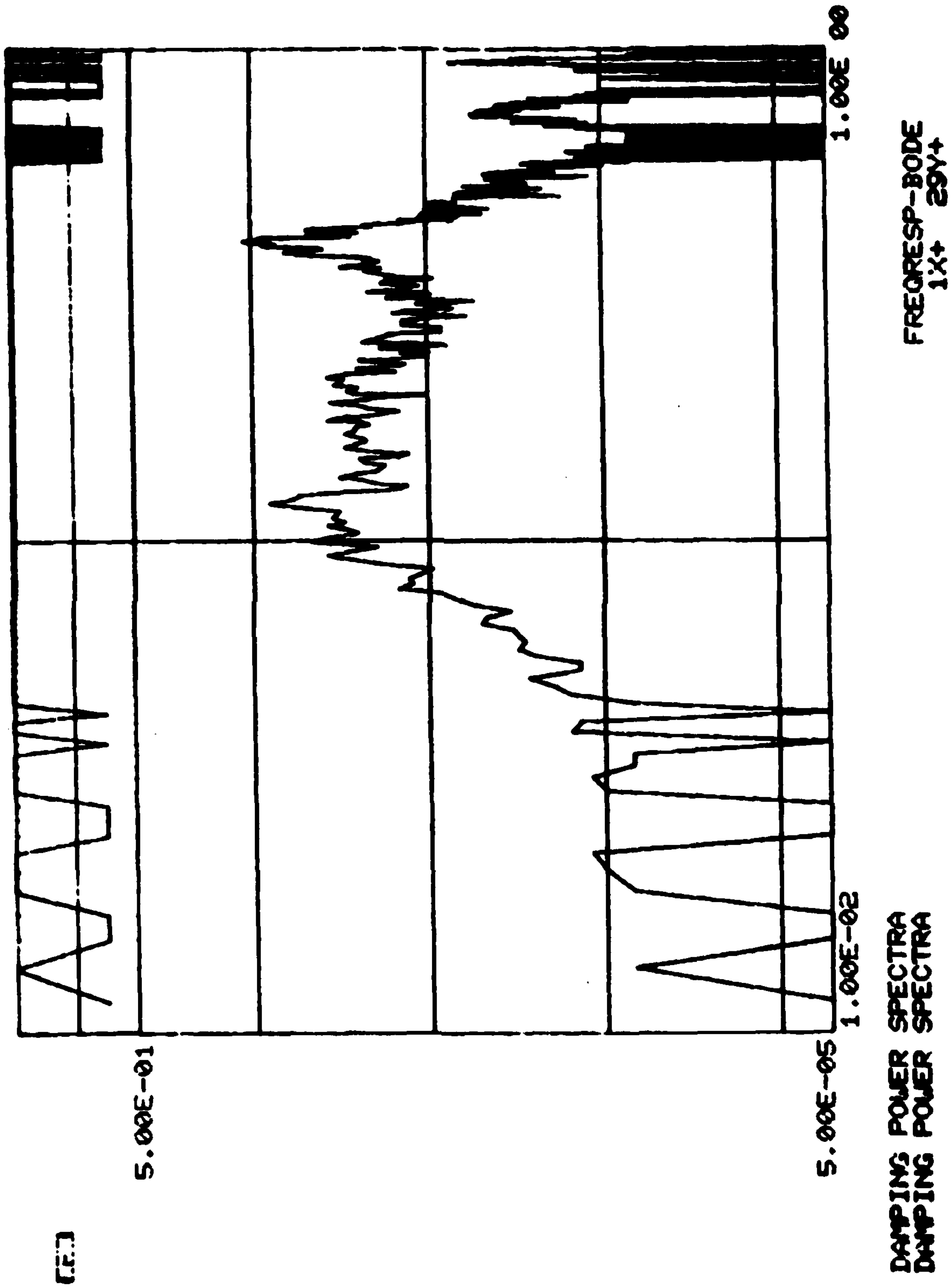
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 28Y+

BLOUGH

ESTIMATED ROOTS (1X+ 29Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.6444E-01	1.000	0.3145E-08	-0.3127E-01
2	0.8995	1.000	0.1344E 07	0.1577E-07
3	0.1109	0.9592E-01	0.8434E-04	0.3064
4	0.2071	0.1252	0.6124E-04	-0.2560
5	0.3147	0.7412E-02	0.1093E-05	2.492
6	0.4033	0.2275E-01	0.2423E-03	-0.2747E-01
7	0.4297	0.3302E-02	0.1022E-04	-0.8053
8	0.5803	0.2241E-02	0.6716E-07	-1.279
9	0.7416	0.2962E-01	0.1404E-05	0.2083
10	0.8324	0.2835E-01	0.1070E-06	2.315
11	0.9428	0.4527E-02	0.5188E-06	0.1072E-02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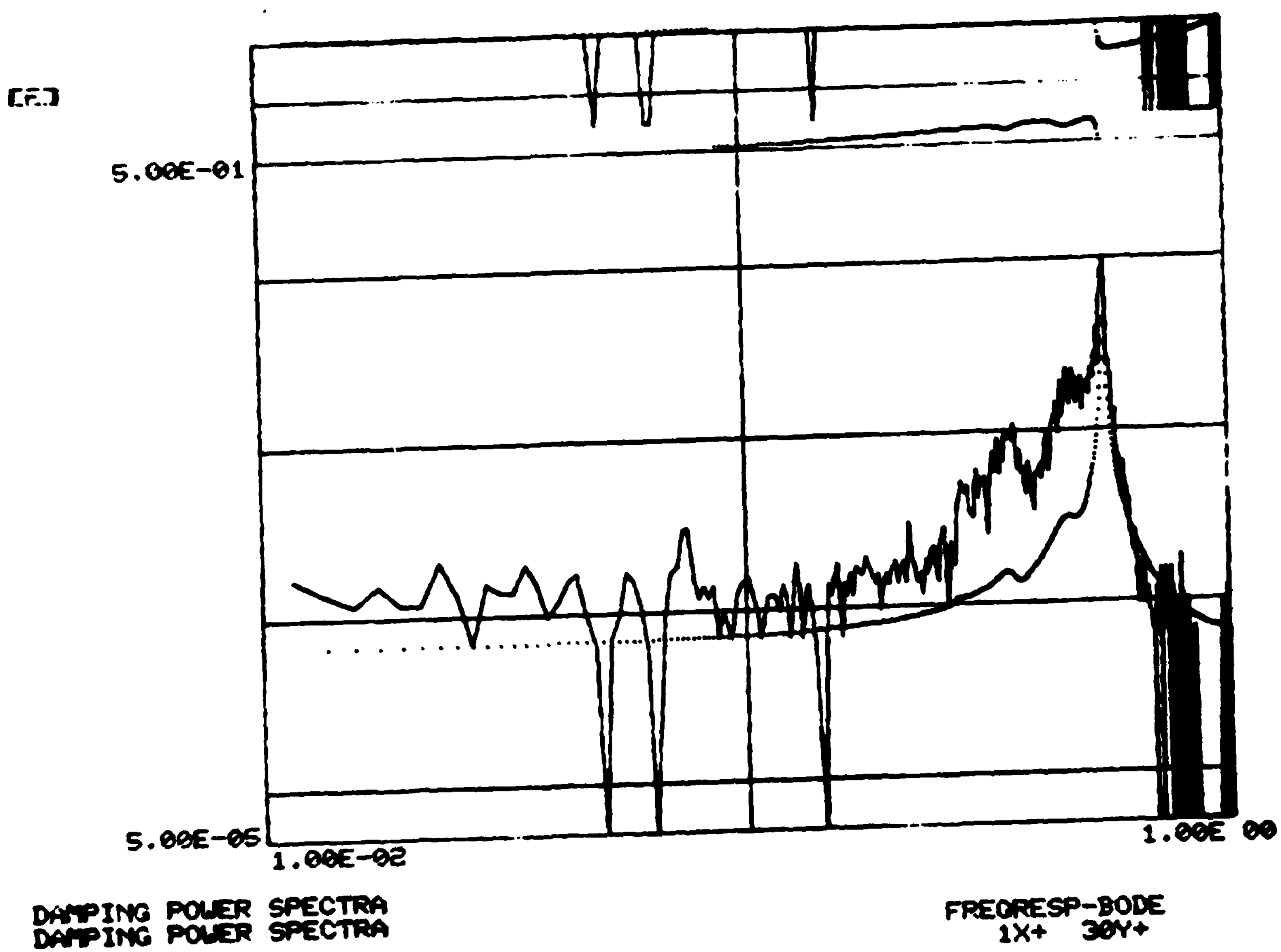
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 29Y+

RYERSON

ESTIMATED ROOTS (1X+ 30Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.6857E-01	1.000	0.3356E-06	3.142
2	0.3850E-01	1.000	0.2825E-09	-3.056
3	0.1125	0.2307	0.1583E-06	-2.043
4	0.2733	0.2755E-01	0.3286E-06	1.336
5	0.3573	0.4472E-01	0.4527E-05	-0.2923
6	0.4660	0.6474E-01	0.2724E-04	0.5120
7	0.5585	0.7445E-02	0.1379E-03	-0.9777E-01
8	0.5608	0.6590E-01	0.3455E-07	2.179
9	0.7250	0.8719E-01	0.2793E-06	0.6665
10	0.8120	0.8558E-02	0.5708E-08	-0.8462
11	1.001	0.4182E-01	0.5984E-07	0.7090E-04
12	1.004	0.8479E-01	0.5570E-11	-1.739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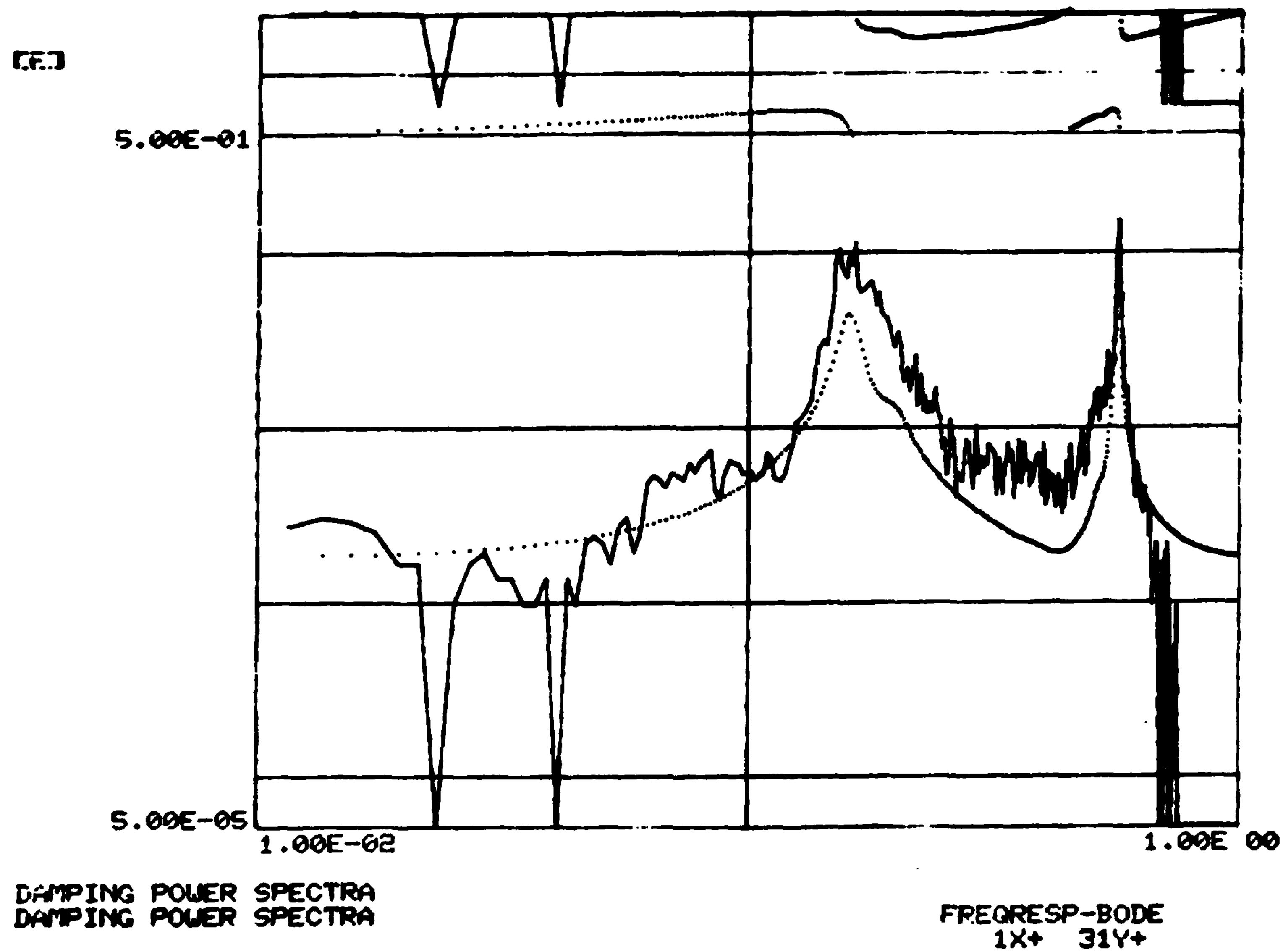
5.00E-05
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 30Y+

RYERSON

ESTIMATED ROOTS (1X+ 31Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1220	0.1219	0.1868E-05	2.089
2	0.1602	0.4601E-01	0.2363E-03	0.1648
3	0.1981	0.7168E-01	0.4687E-04	-0.7732
4	0.2397	0.1488	0.1203E-06	0.9315
5	0.4022	0.7111E-01	0.2194E-05	-1.519
6	0.5214	0.2390E-01	0.7457E-05	-0.1919
7	0.5654	0.4556E-02	0.1751E-03	-0.1188E-01
8	0.6810	0.1241	0.9816E-06	2.827
9	0.7073	0.5858E-03	0.4012E-07	0.2913
10	0.9359	0.2532E-01	0.1803E-08	0.7178

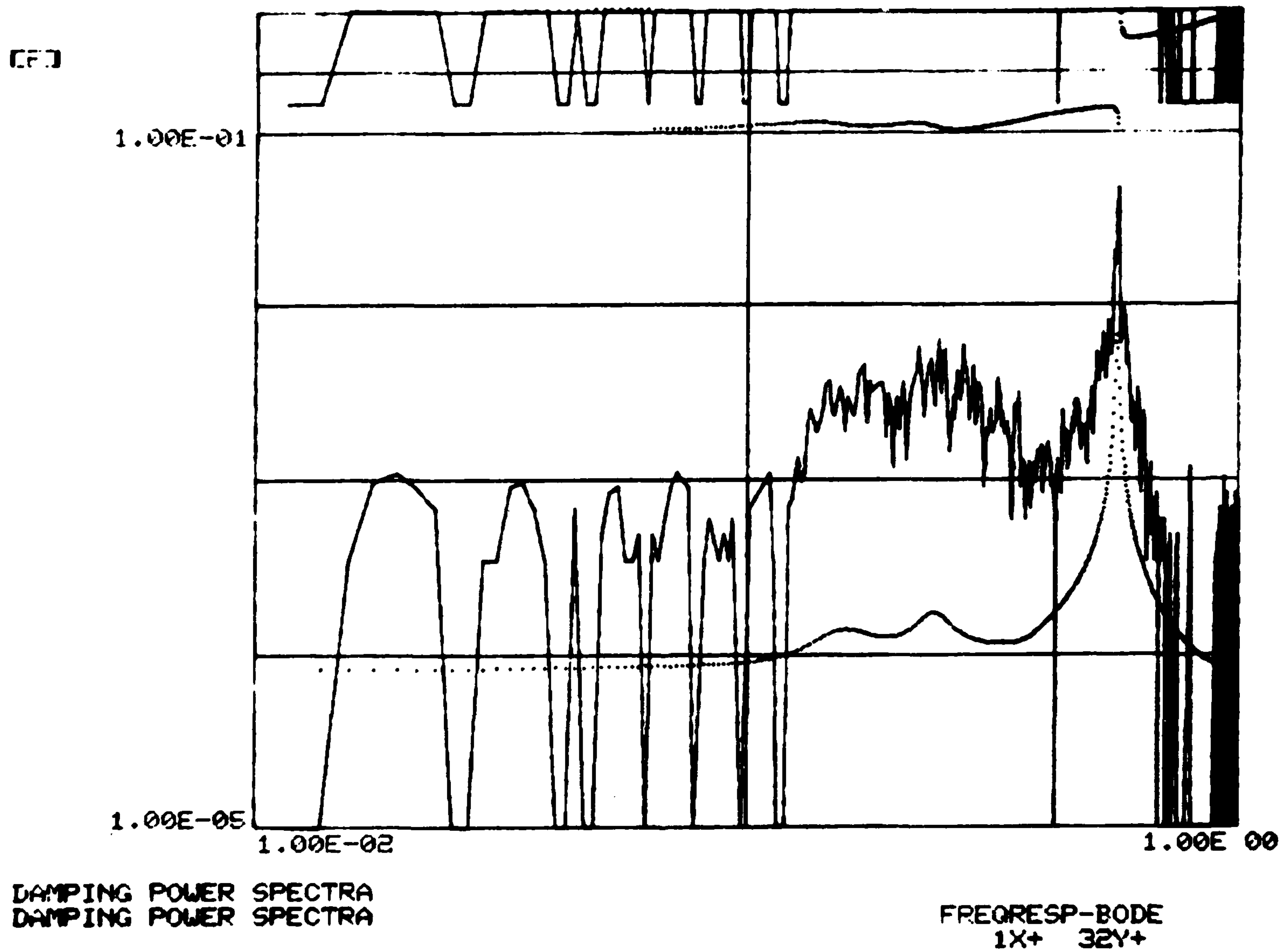


[F]
5.00E-01
5.00E-05
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 31Y+

RYERSON

ESTIMATED ROOTS (1X+ 32Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1482	1.000	0.4640E-05	0.2293E-05
2	0.1052	1.000	0.2440E-05	-3.142
3	0.1546	0.1733	0.9310E-06	0.8281
4	0.2392	0.9447E-01	0.1150E-05	0.5128
5	0.2832	0.1519	0.1758E-08	0.4183
6	0.3847	0.1428	0.6377E-06	-2.655
7	0.5230	0.5615E-01	0.1807E-07	-0.6959
8	0.5661	0.6254E-02	0.1790E-04	-0.3040E-01
9	0.6344	0.1621E-01	0.5290E-07	-0.9352
10	0.7965	0.5173E-02	0.3034E-03	0.2686
11	0.9253	0.1889E-01	0.1769E-07	0.7029



[F]
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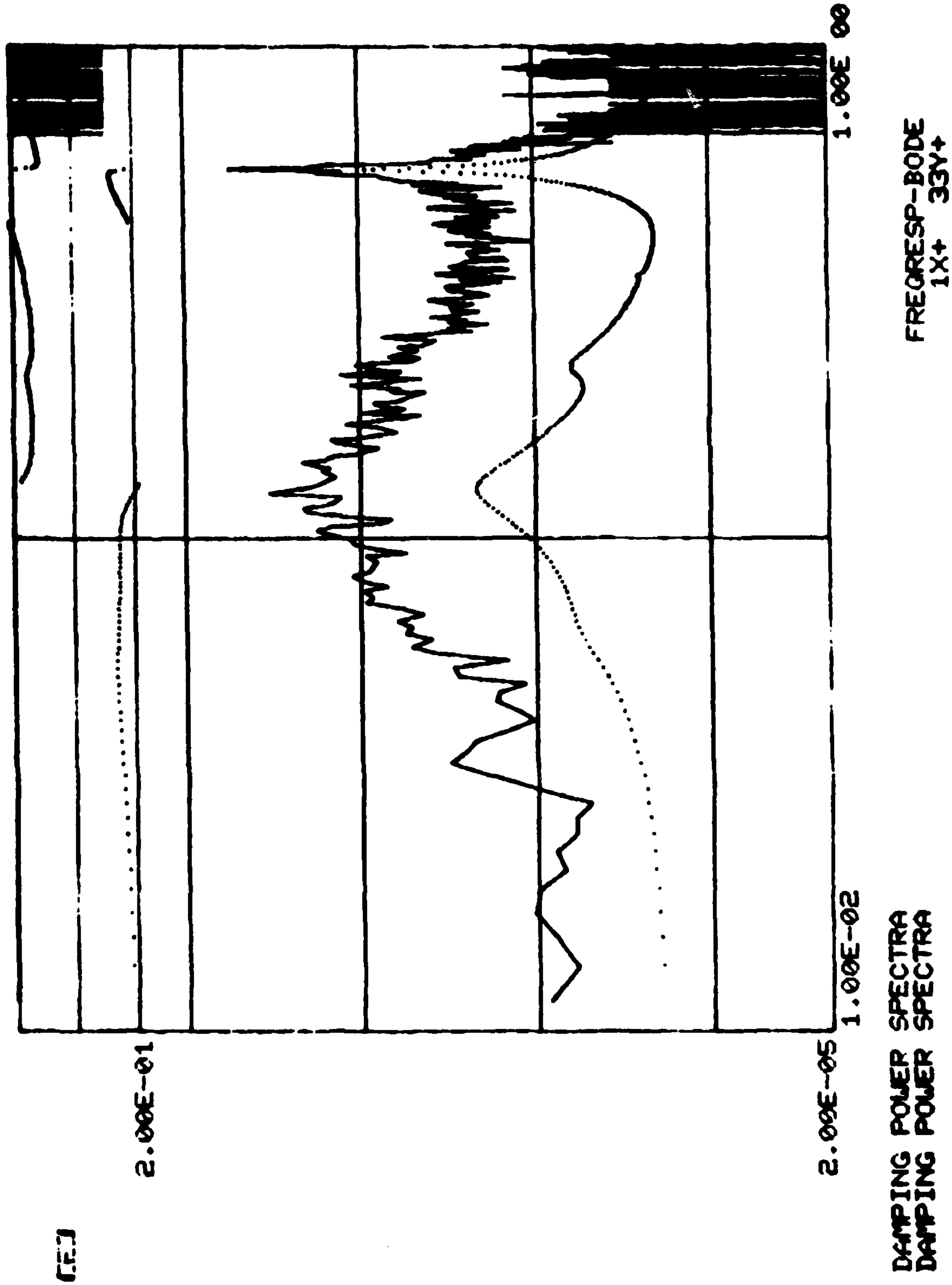
1.00E-05
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 32Y+

RYERSON

ESTIMATED ROOTS (1X+ 33Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.6421E-01	0.2180	0.1336E-05	1.429
2	0.1266	0.1224	0.2274E-04	0.3651E-01
3	0.2226	0.7349E-01	0.2828E-05	-0.2597
4	0.3439	0.8330E-02	0.3784E-07	-0.9551
5	0.4413	0.3117E-01	0.2430E-07	-0.4055
6	0.5640	0.4955E-02	0.1905E-04	-0.3385E-01
7	0.5910	0.5866E-01	0.7688E-08	-1.340
8	0.7746	0.3759E-01	0.7851E-09	1.693
9	0.8987	0.4498E-02	0.3051E-07	0.8140E-01
10	0.9389	0.6692E-01	0.3576E-10	-0.8202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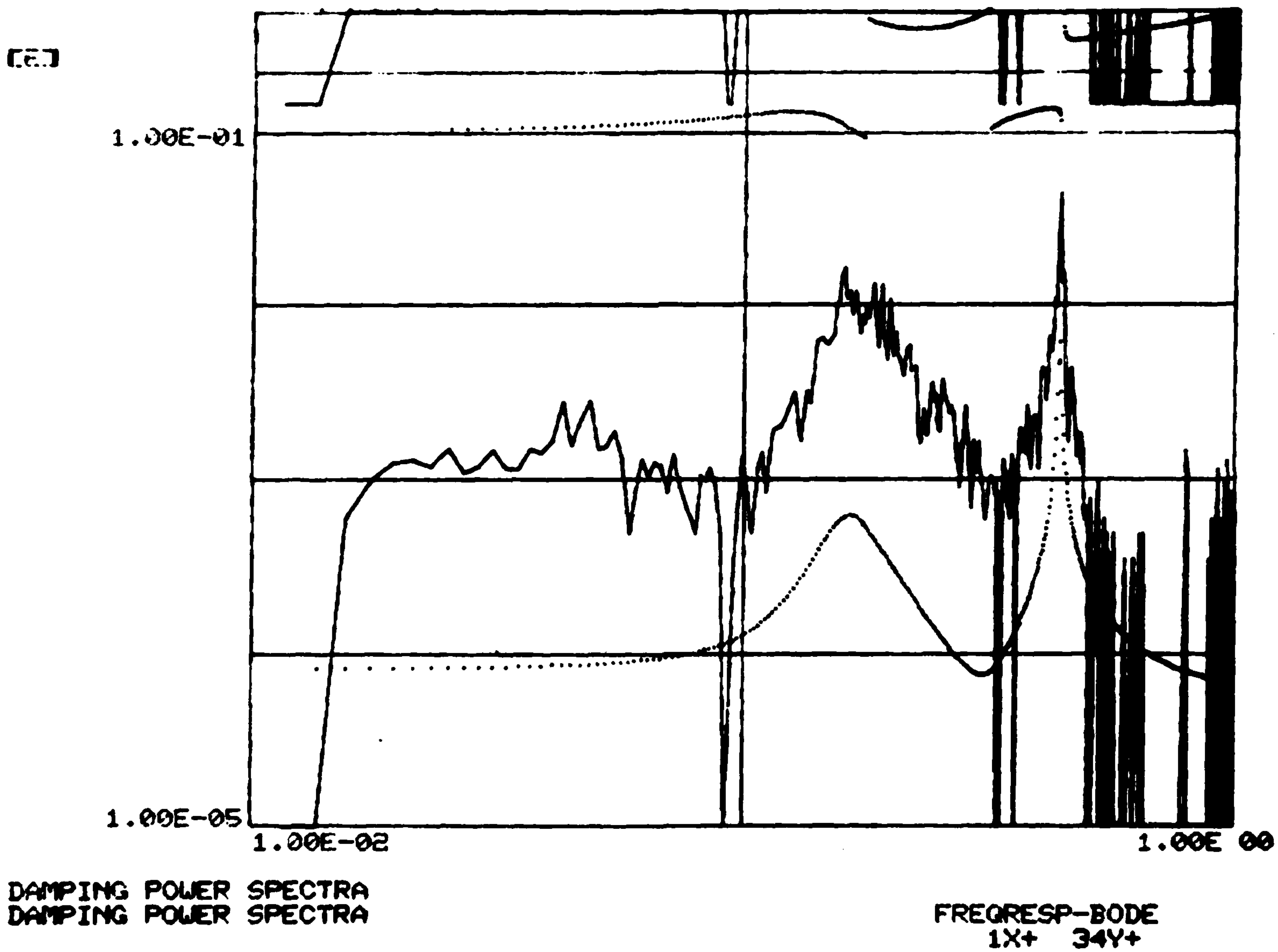
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 33Y+

RYERSON

ESTIMATED ROOTS (1X+ 34Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.5915E-03	1.000	0.1011E-06	3.142
2	0.1606	0.3937	0.5849E-05	-1.958
3	0.1633	0.1307	0.1058E-04	0.4553
4	0.2213	0.8120E-01	0.6647E-07	-1.976
5	0.3685	0.2306	0.8730E-06	2.362
6	0.4395	0.5237E-02	0.1080E-04	-0.3632E-01
7	0.4859	0.2046	0.1245E-11	-2.747
8	0.6522	0.3867E-02	0.1247E-08	-0.3478
9	0.7986	0.3890E-02	0.9027E-08	0.9337E-01
10	0.9108	0.6005E-01	0.4175E-07	1.441
11	1.001	0.4252E-01	0.9949E-10	3.139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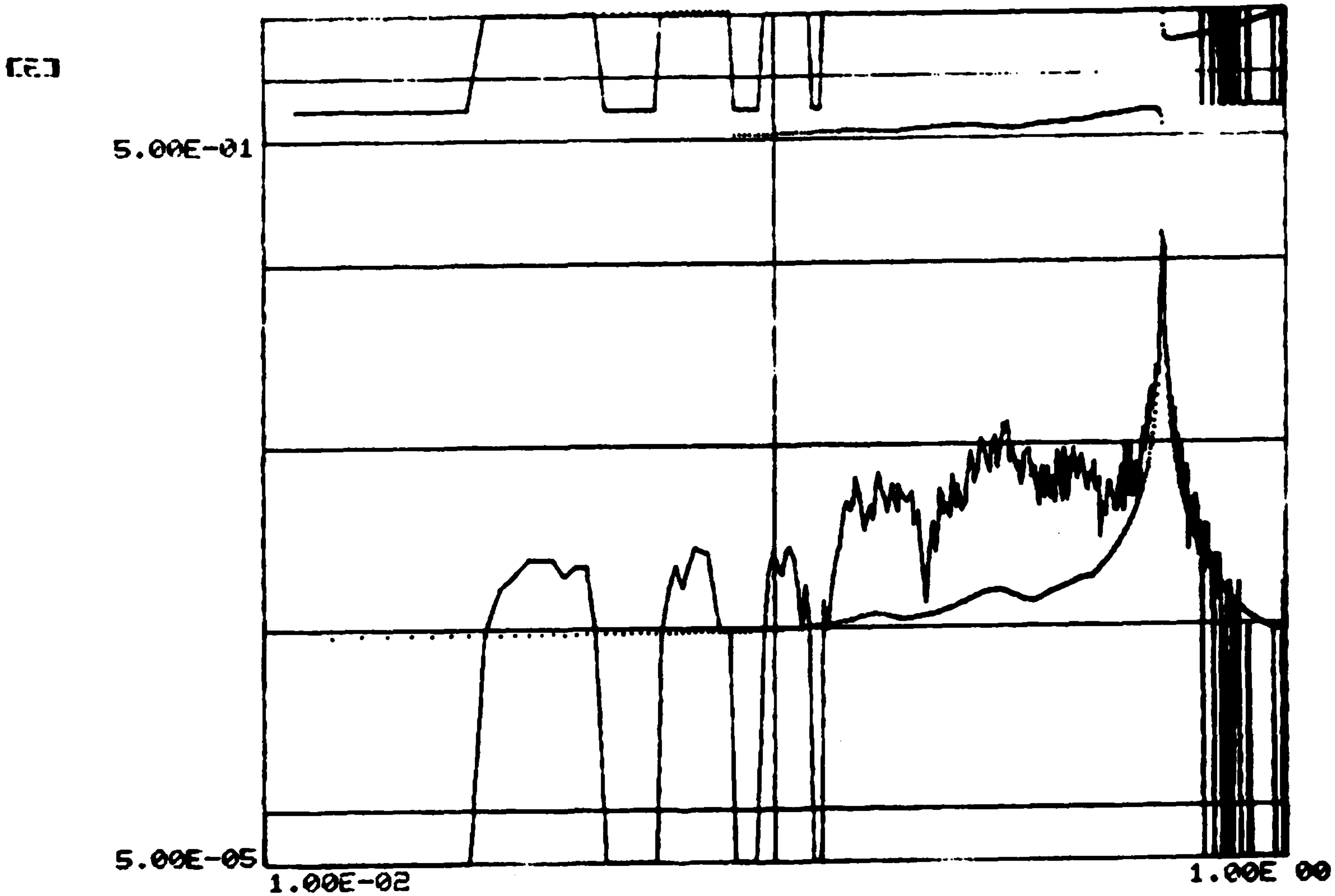
1.00E-05
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 34Y+

FYERSON

ESTIMATED ROOTS (1X+ 35Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1770	1.000	0.7171E-05	-0.5569E-05
2	0.2586E-01	1.000	0.4216E-06	3.142
3	0.1587	0.1032	0.1804E-05	0.1897
4	0.2746	0.1545	0.1542E-04	0.4699
5	0.3335	0.8548E-01	0.3661E-05	2.582
6	0.4170	0.3591E-01	0.1558E-05	-1.401
7	0.5714	0.3690E-01	0.8193E-06	1.846
8	0.5760	0.4053E-02	0.2121E-03	-0.1612E-01
9	0.7440	0.1893	0.1957E-05	-2.295
10	0.7568	0.1214E-01	0.6774E-07	2.320
11	1.000	0.4894E-02	0.3000E-07	0.1345E-03
12	1.002	0.7022E-01	0.1729E-10	-0.2902E-01



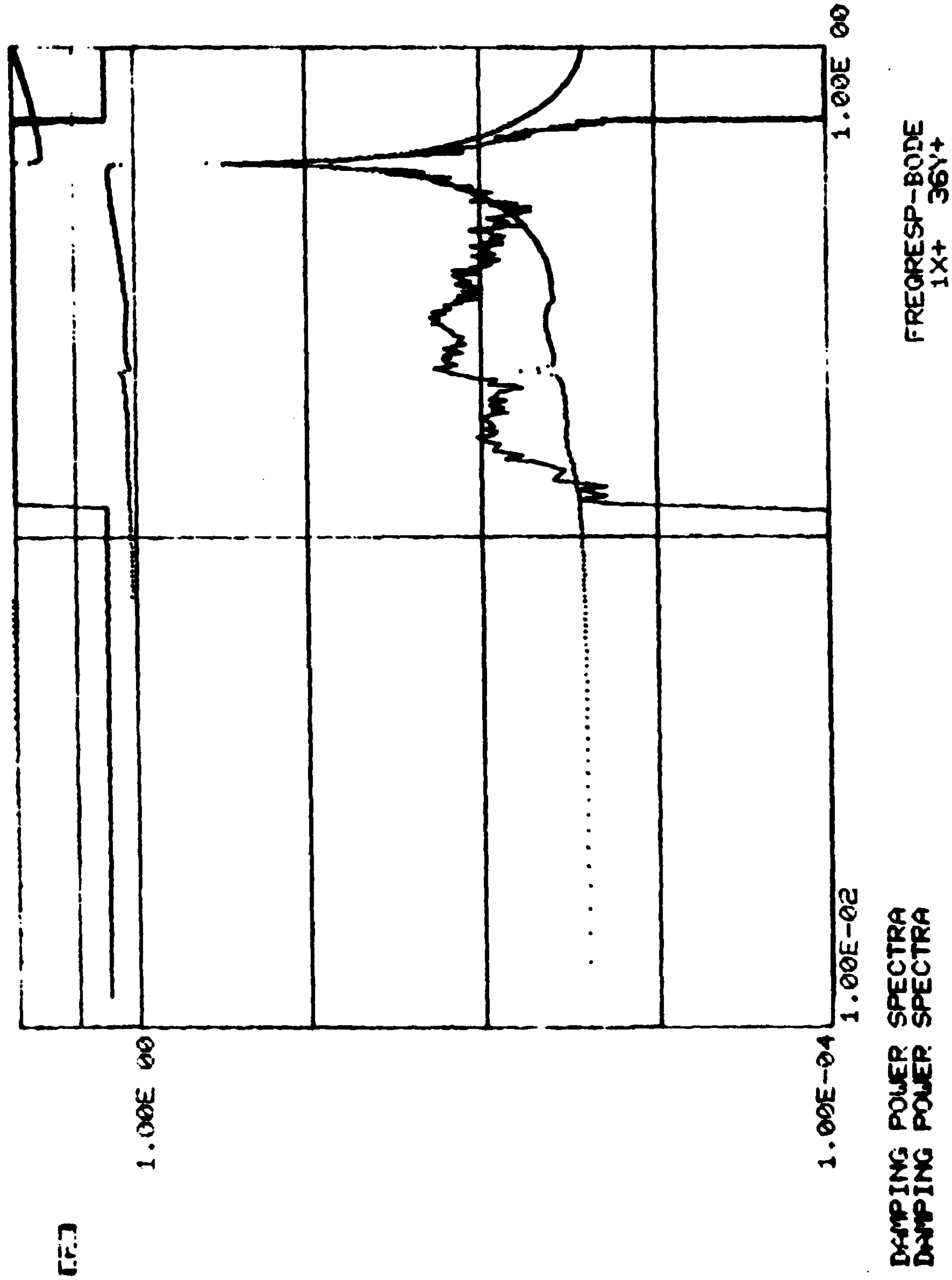
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 35Y+

RYERSON

ESTIMATED ROOTS (1X+ 36Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1542	0.1227	0.4329E-05	0.8964
2	0.2187	0.7493E-02	0.3026E-05	0.6333
3	0.2835	0.1790	0.3730E-04	0.1394
4	0.3042	0.2247E-01	0.1184E-05	-2.277
5	0.4615	0.4297E-01	0.2532E-05	-2.763
6	0.5382	0.1545E-01	0.1077E-05	-1.211
7	0.5798	0.3627E-02	0.5942E-03	-0.1182E-01
8	0.6982	0.3798E-01	0.9063E-06	-2.176
9	0.7131	0.1897	0.1138E-05	2.474
10	0.9204	0.7668E-03	0.1388E-08	0.7787

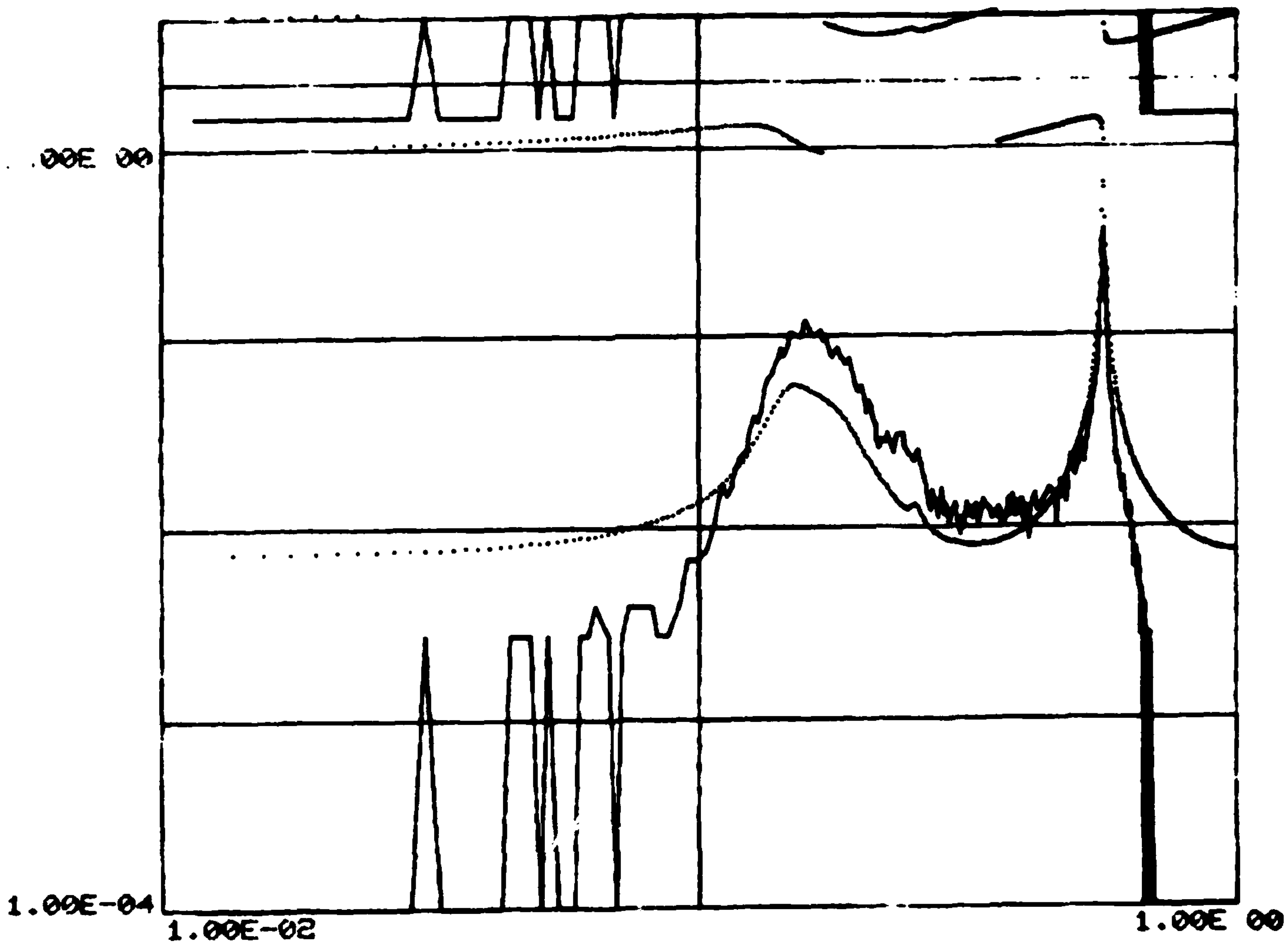


1.00E 00
1.00E-04
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 36Y+

RYERSON

ESTIMATED ROOTS (1X+ 37Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.3433E-01	1.000	0.1225E-05	3.140
2	0.1473	0.1073	0.4472E-03	0.9253
3	0.1810	0.1523	0.4723E-03	-0.8685
4	0.2526	0.2470E-01	0.1159E-04	-0.3252
5	0.3313	0.3729	0.3861E-04	0.9460
6	0.4253	0.7230E-01	0.1623E-05	2.868
7	0.5672	0.4329E-02	0.1231E-02	-0.2109E-01
8	0.5695	0.3237E-01	0.3670E-05	2.801
9	0.6632	0.8341E-01	0.5389E-05	-3.051
10	0.9187	0.5606E-02	0.2165E-08	1.413
11	1.000	0.3064E-02	0.4777E-09	-3.073



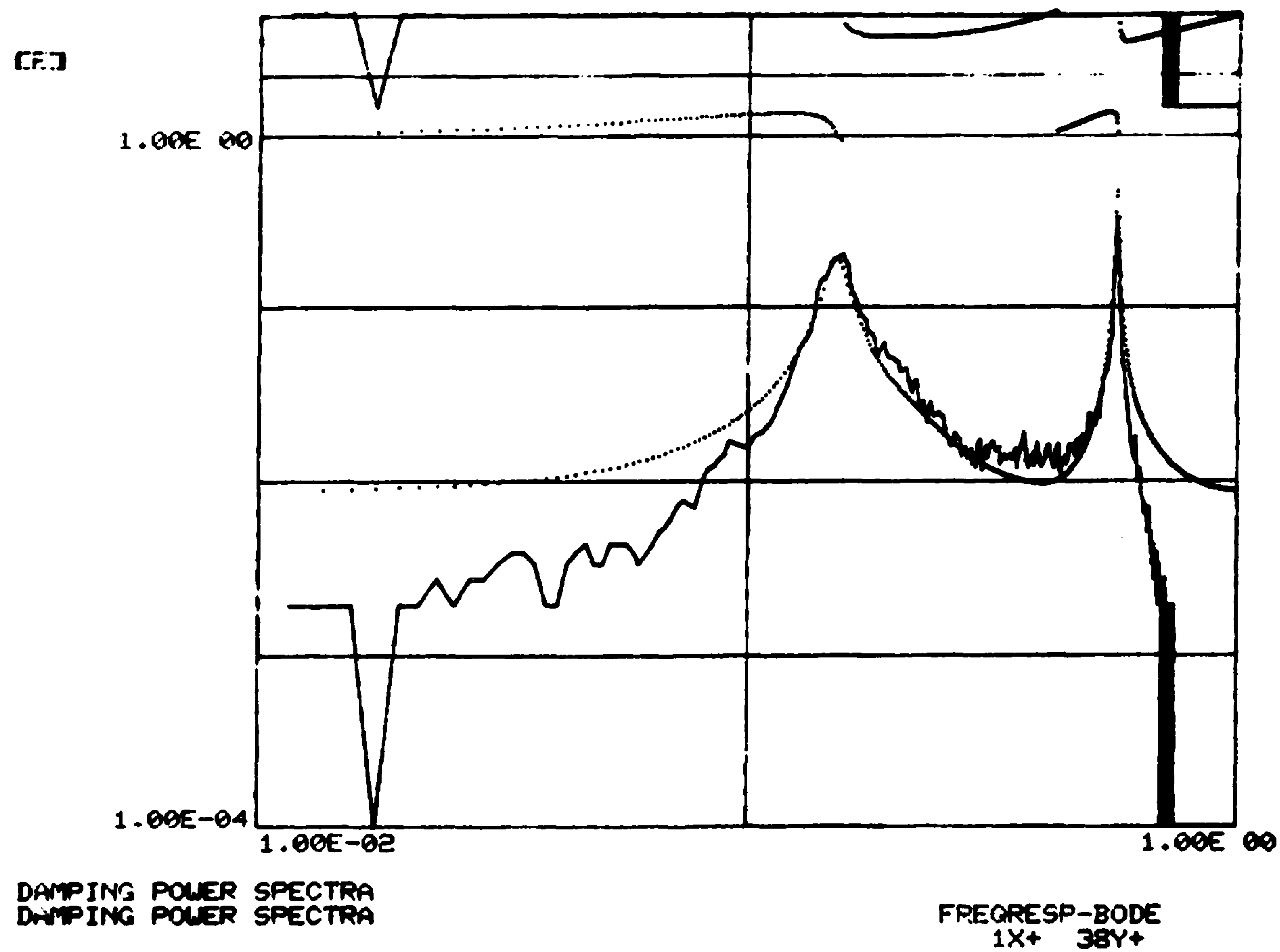
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 37Y+

RYERSON

ESTIMATED ROOTS (1X+ 38Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1704	1.000	0.1007E-03	-0.1383E-03
2	0.1084	0.2445	0.3154E-06	1.755
3	0.1515	0.5208E-01	0.1117E-02	0.1402
4	0.2372	0.1955	0.8562E-04	-2.063
5	0.3895	0.1123	0.7093E-05	-1.090
6	0.5062	0.1011	0.6584E-08	-1.340
7	0.5671	0.5139E-02	0.1045E-02	-0.2892E-01
8	0.6512	0.7877E-01	0.2251E-05	-2.418
9	0.7385	0.2014E-02	0.2452E-07	2.577
10	0.9761	0.1293	0.4723E-06	2.427
11	1.131	0.4673	0.5302E-13	-3.142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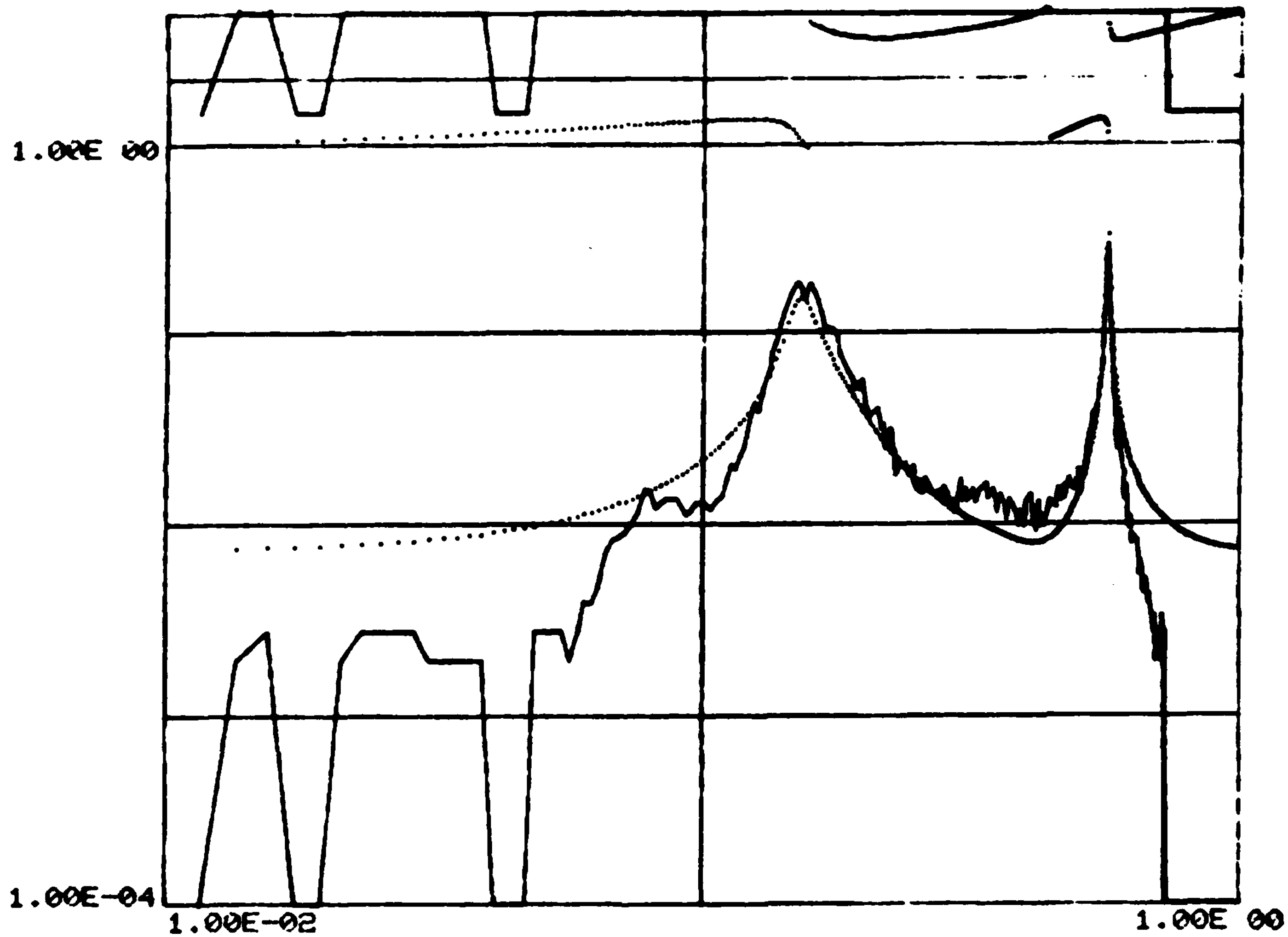
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 38Y+

RYERSON

ESTIMATED ROOTS (1X+ 39Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1262	0.2927	0.3947E-06	0.8336
2	0.1522	0.6183E-01	0.1013E-02	0.3047
3	0.1829	0.1757	0.2860E-03	-1.468
4	0.3367	0.1369	0.1616E-04	0.2529E-01
5	0.4854	0.6416E-01	0.1671E-06	-1.840
6	0.5598	0.1376	0.1391E-04	1.828
7	0.5666	0.5784E-02	0.8025E-03	-0.3674E-01
8	0.7471	0.6076E-01	0.4347E-06	2.858
9	0.8903	0.1745	0.1814E-09	-2.348
10	1.000	0.3901E-02	0.3647E-08	3.137
11	1.032	0.2473	0.1271E-05	0.9955E-03



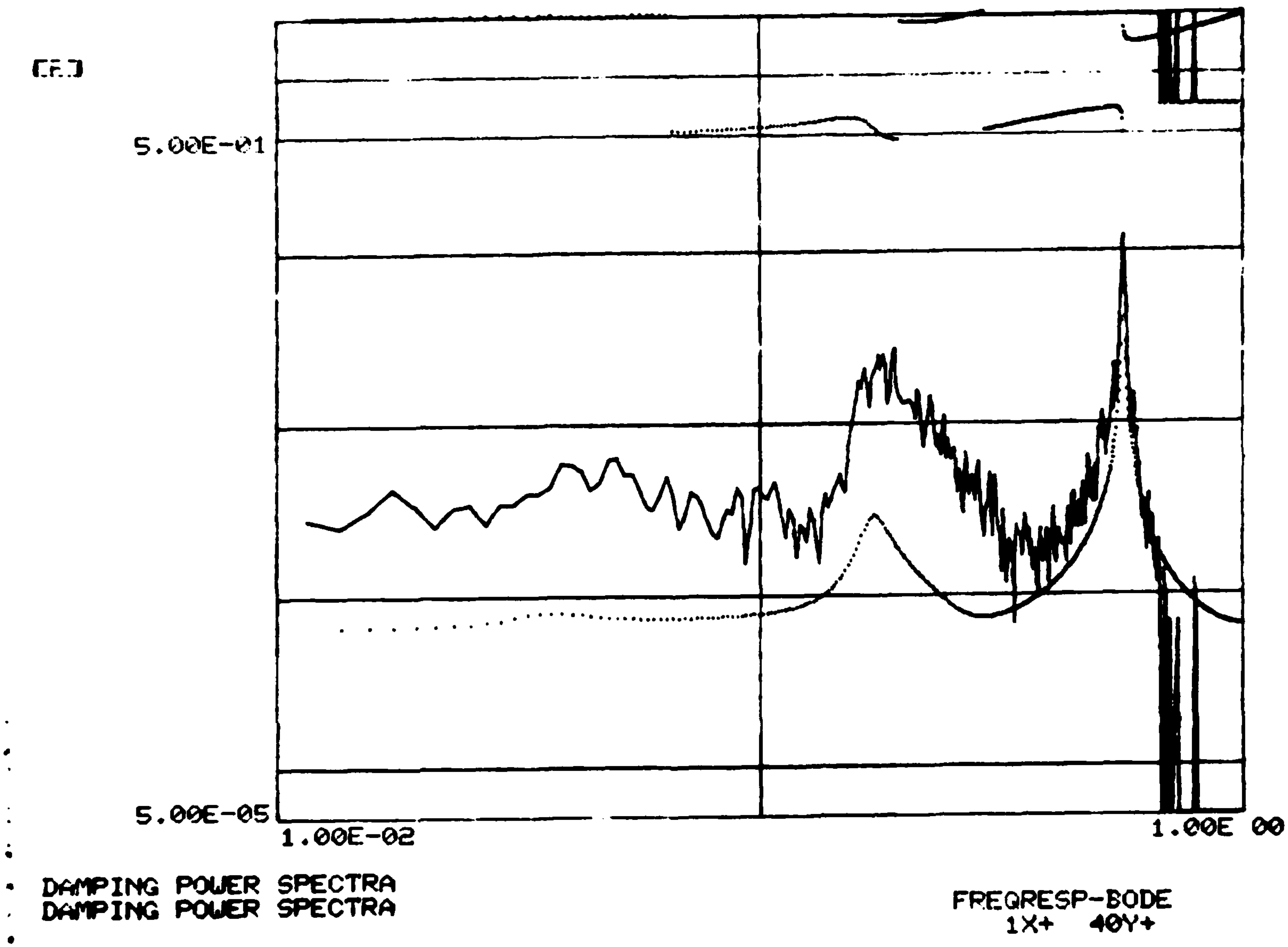
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 39Y+

RYERSON

ESTIMATED ROOTS (1X+ 40Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.3580E-01	0.2411	0.7807E-06	0.7170
2	0.1712	0.7069E-01	0.1893E-04	0.6343
3	0.2153	0.2599	0.1618E-04	-1.091
4	0.2811	0.7376E-01	0.3217E-07	2.020
5	0.5112	0.4525E-01	0.2372E-05	0.5571
6	0.5666	0.4218E-02	0.1380E-03	-0.4884E-02
7	0.5978	0.1274E-01	0.1268E-05	-1.902
8	0.6766	0.1152	0.8121E-06	2.724
9	0.7993	0.2132E-01	0.1040E-08	0.1278
10	1.008	0.1240	0.1008E-06	0.1468E-03
11	1.001	0.5141E-01	0.1169E-07	-3.141



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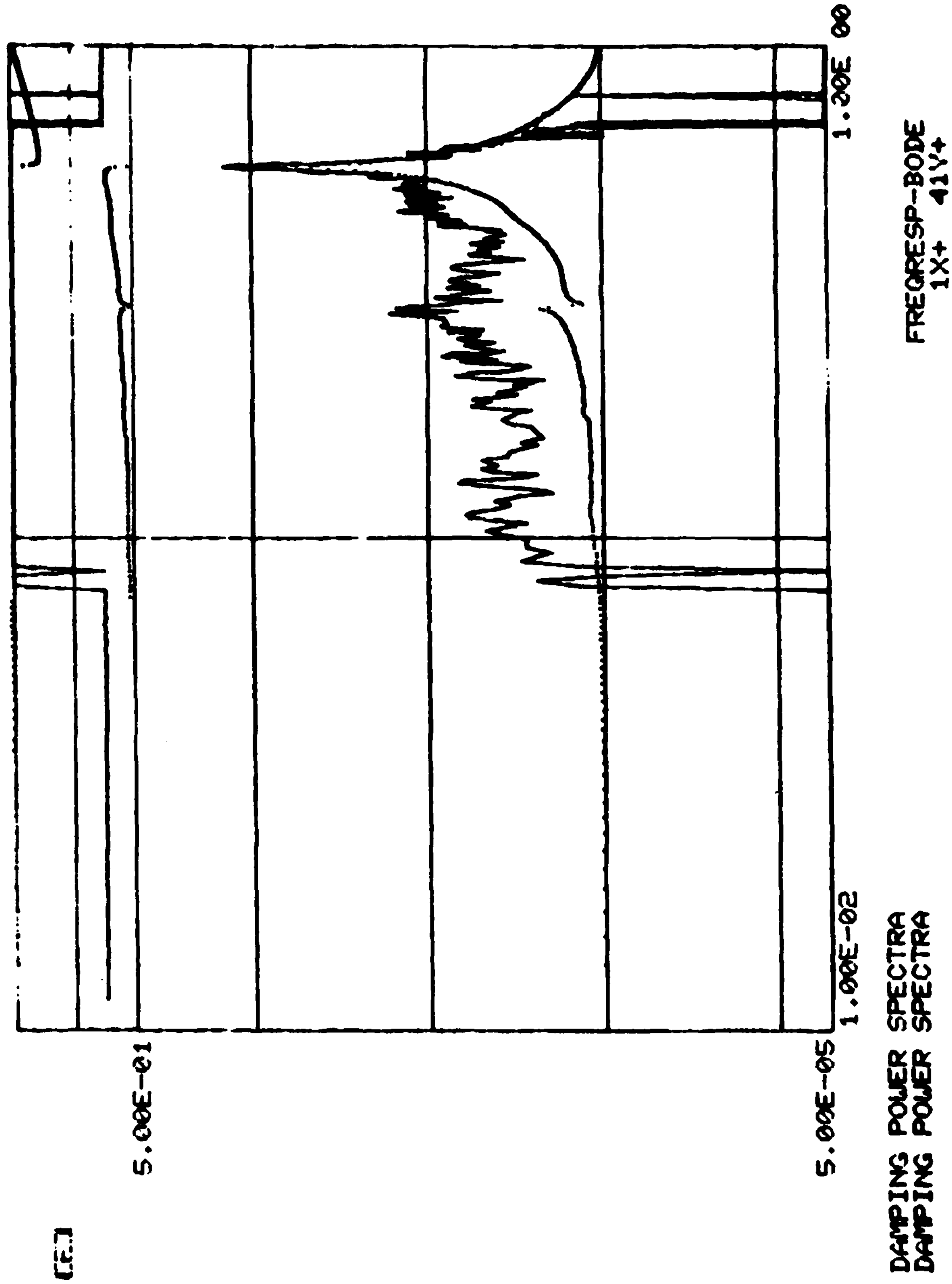
1.00E-02
1.00E 00
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 40Y+

RYERSON

ESTIMATED ROOTS (1X+ 41Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1092	0.1828	0.1224E-05	0.3192
2	0.1763	0.2119E-01	0.1922E-06	1.905
3	0.2950	0.9471E-02	0.2057E-05	-0.7606
4	0.3194	0.1533	0.9014E-05	0.5451E-01
5	0.4573	0.7077E-01	0.8807E-05	0.8621
6	0.5472	0.1089	0.3032E-08	0.1052
7	0.5690	0.3873E-02	0.2382E-03	-0.2798E-01
8	0.6803	0.8105E-01	0.6439E-06	-2.614
9	0.8010	0.3520E-01	0.1657E-07	-0.6210
10	1.004	0.9401E-01	0.1836E-10	-1.864
11	1.000	0.8280E-02	0.2986E-09	3.122



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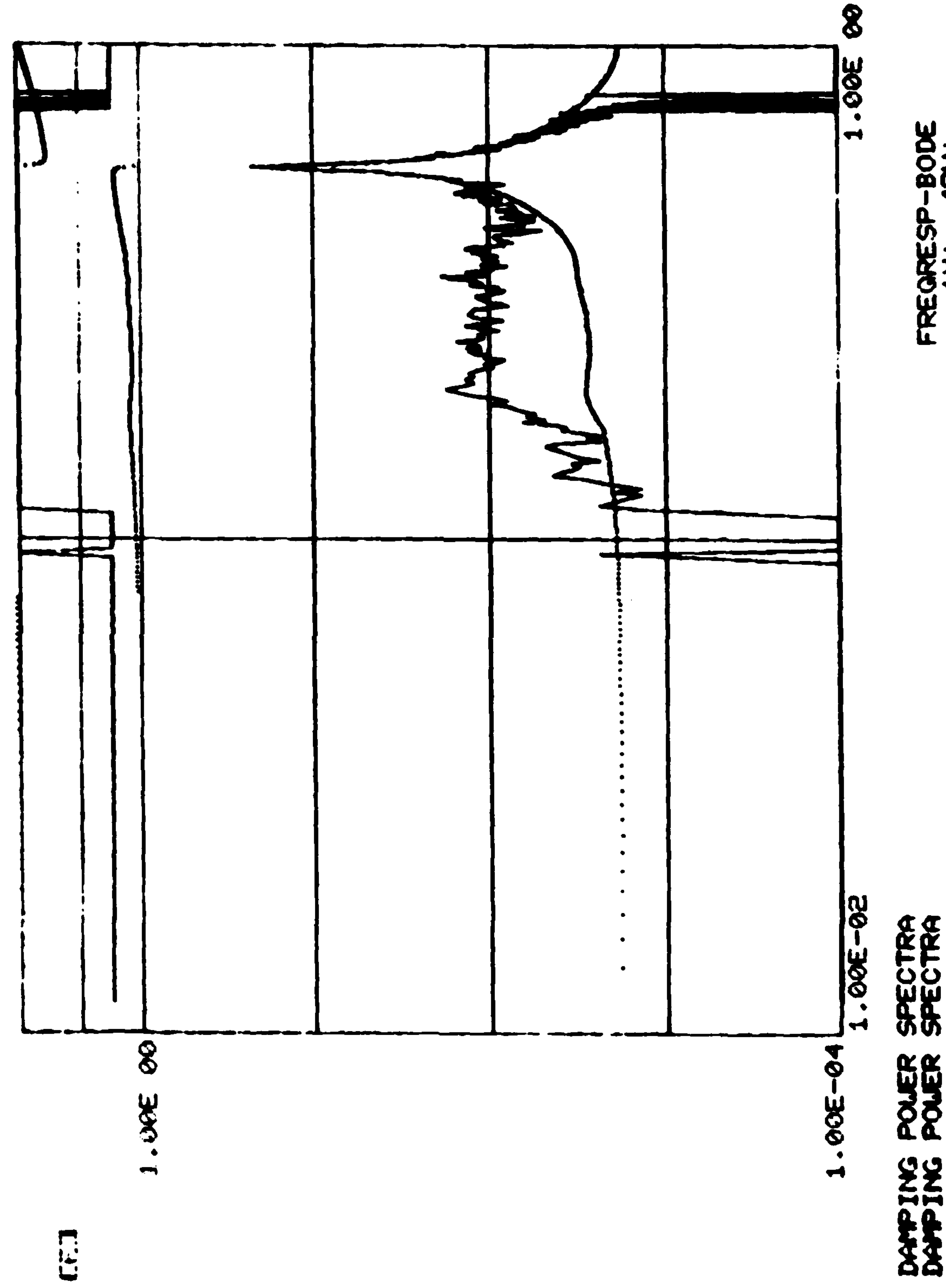
1.20E 00
FREQRESP-BODE
1X+ 41Y+
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

RYERSON

ESTIMATED ROOTS (1X+ 42Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.4352	1.000	0.1055E-10	0.7468E-07
2	0.1687	0.3575E-01	0.5409E-06	-3.023
3	0.1969	0.1578	0.1352E-04	0.7618
4	0.2826	0.3796E-01	0.5233E-06	-1.236
5	0.3511	0.1867	0.2177E-04	-0.2536
6	0.4484	0.1822E-01	0.4657E-06	2.817
7	0.5488	0.4672E-01	0.8041E-06	0.7784
8	0.5672	0.4758E-02	0.4177E-03	-0.1012E-01
9	0.7451	0.1323	0.1671E-05	-2.655
10	0.7530	0.2510E-01	0.1294E-07	-1.768
11	1.000	0.8120E-02	0.1631E-08	3.141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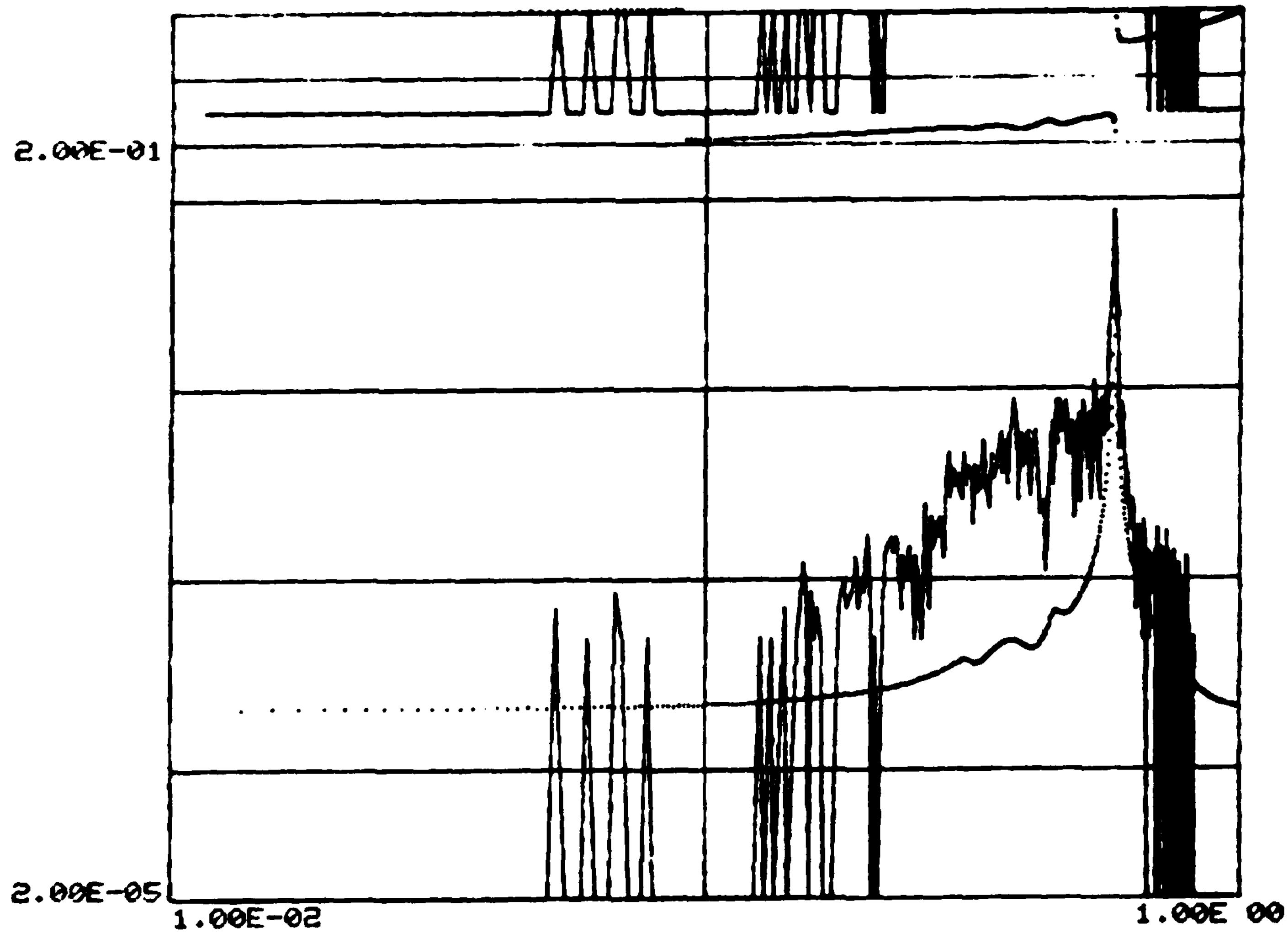
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 42Y+

RYERSON

ESTIMATED ROOTS (1X+ 43Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.8738E-01	0.7745E-01	0.7376E-08	-1.533
2	0.2429	0.3248E-01	0.1215E-07	-2.646
3	0.3125	0.4195E-01	0.5246E-06	-0.5387
4	0.3827	0.9956E-01	0.4670E-05	0.1315
5	0.4486	0.3819E-01	0.2480E-05	1.238
6	0.5182	0.4701E-01	0.4842E-07	0.4998
7	0.5816	0.3593E-02	0.4530E-04	-0.4930E-01
8	0.7800	0.1616	0.5122E-06	-0.3373
9	0.7831	0.4954E-01	0.1416E-09	2.821
10	0.7986	0.9712E-01	0.2489E-06	-2.752



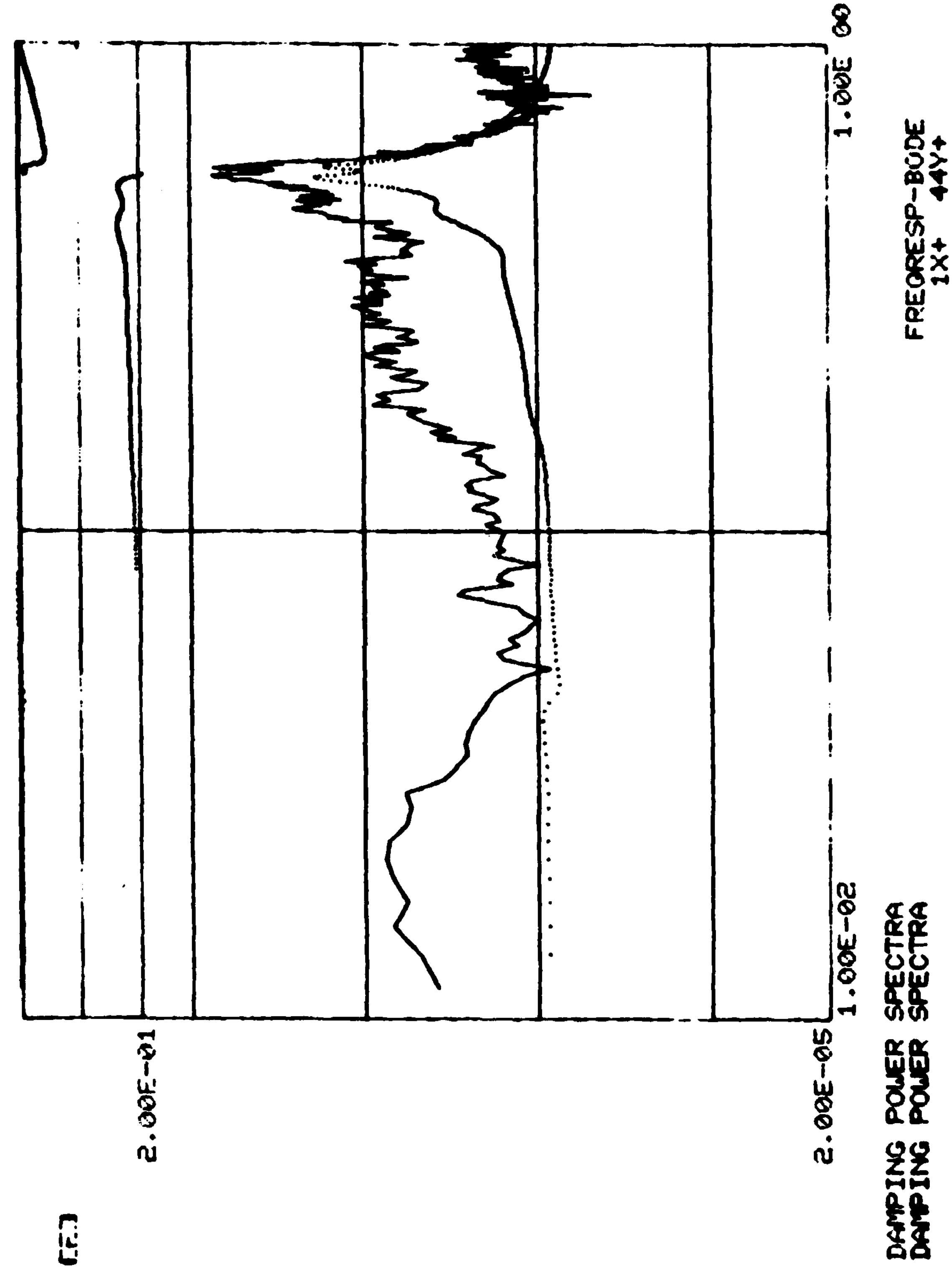
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 43Y+

RYERSON

ESTIMATED ROOTS (1X+ 44Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.4530E-01	0.8182E-01	0.5489E-06	-1.547
2	0.1687	0.1842	0.3099E-05	1.644
3	0.2982	0.6662E-01	0.1979E-06	-0.8891
4	0.3665	0.1489	0.1750E-04	-0.8943
5	0.4558	0.5646E-01	0.3028E-04	0.5573
6	0.5320	0.1584E-01	0.9968E-04	0.1625
7	0.5607	0.1248E-01	0.6319E-04	-0.4434
8	0.7835	0.1922E-02	0.2182E-07	-0.2649
9	0.9780	0.1425E-01	0.4575E-07	-3.093
10	1.011	0.1454	0.3000E-05	-0.3725E-05
11	1.003	0.7593E-01	0.1565E-09	-2.919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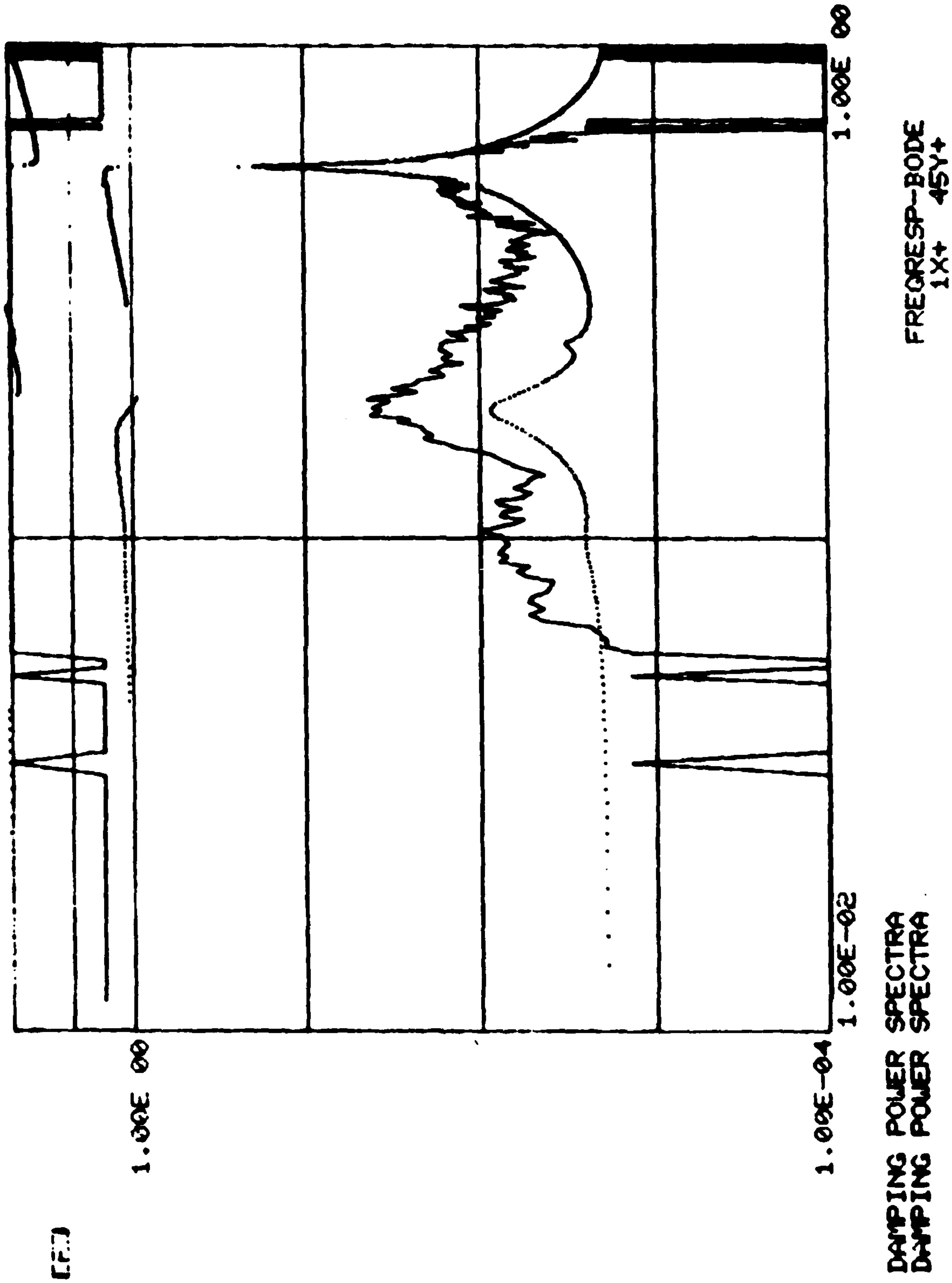
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 44Y+

RYERSON

ESTIMATED ROOTS (1X+ 45Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.7953E-01	1.000	0.4375E-05	0.2567E-04
2	0.1037	0.1632	0.4429E-05	-0.2260
3	0.1827	0.7481E-01	0.6977E-04	0.2185
4	0.2475	0.3031E-01	0.3751E-05	0.1168
5	0.4545	0.5377E-01	0.5455E-05	1.468
6	0.5210	0.2931E-02	0.2275E-05	-2.007
7	0.5682	0.3952E-02	0.3996E-03	-0.2281E-01
8	0.6410	0.3603E-01	0.6614E-06	-1.582
9	0.7203	0.3112E-01	0.8308E-07	-2.698
10	0.9471	0.2097E-01	0.6535E-07	1.082
11	1.000	0.7289E-02	0.1787E-07	-0.3360E-02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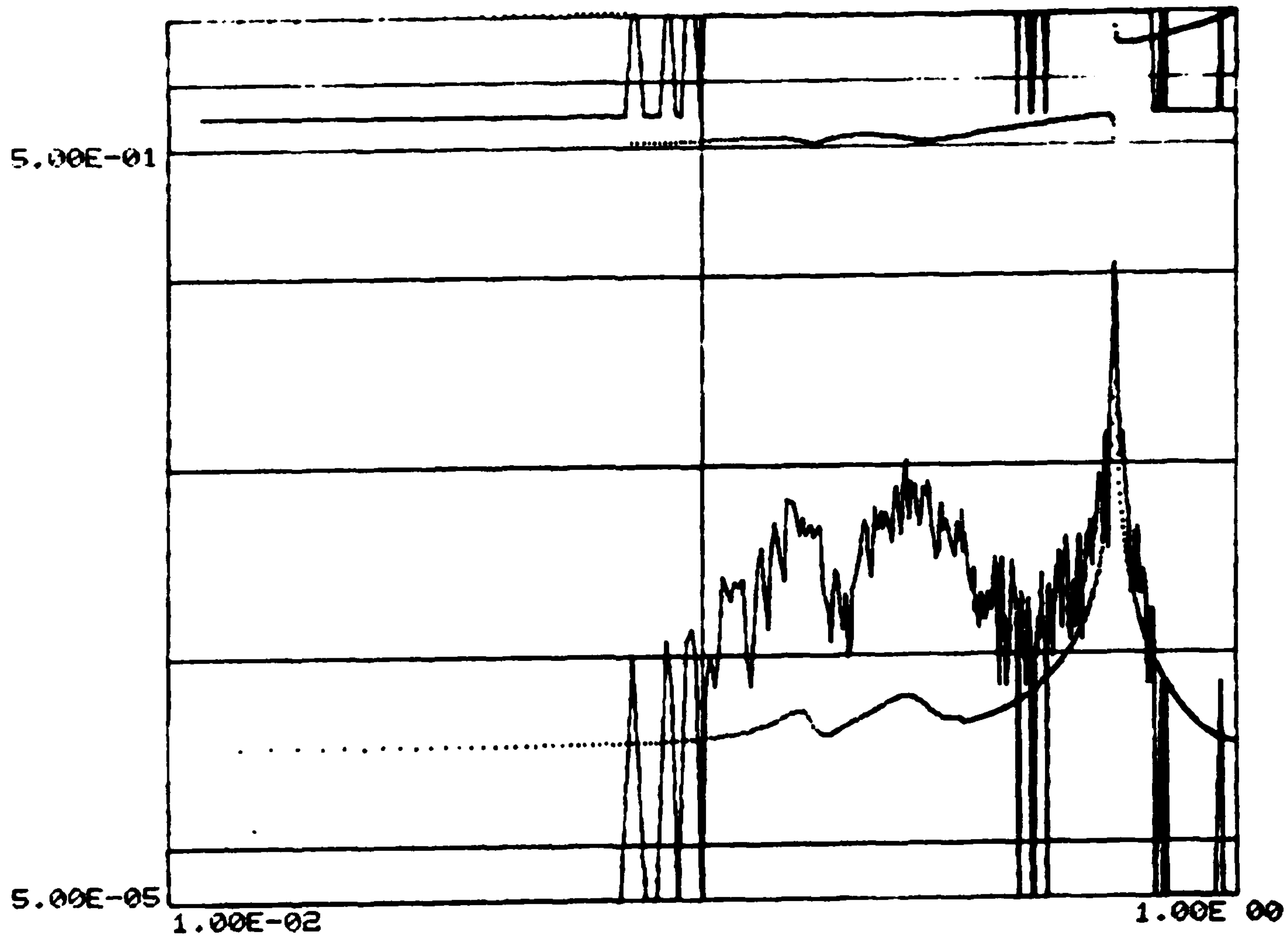
1.00E 00
FREQRESP-BODE
1X+ 45Y+
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

RYERSON

ESTIMATED ROOTS (1X+ 46Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1619	0.3698	0.1894E-05	2.399
2	0.1580	0.6666E-01	0.1330E-05	-0.7255
3	0.2486	0.1243	0.5271E-05	-0.4730E-01
4	0.3044	0.1985E-01	0.1142E-06	-0.3122
5	0.4308	0.1712E-01	0.1021E-06	2.388
6	0.5244	0.2137E-01	0.5455E-07	1.770
7	0.5922	0.3365E-02	0.7939E-04	-0.7576E-02
8	0.6607	0.1200	0.8671E-06	2.948
9	0.6774	0.1390E-02	0.6638E-07	-0.2225
10	0.9352	0.6213E-02	0.1550E-08	0.1610



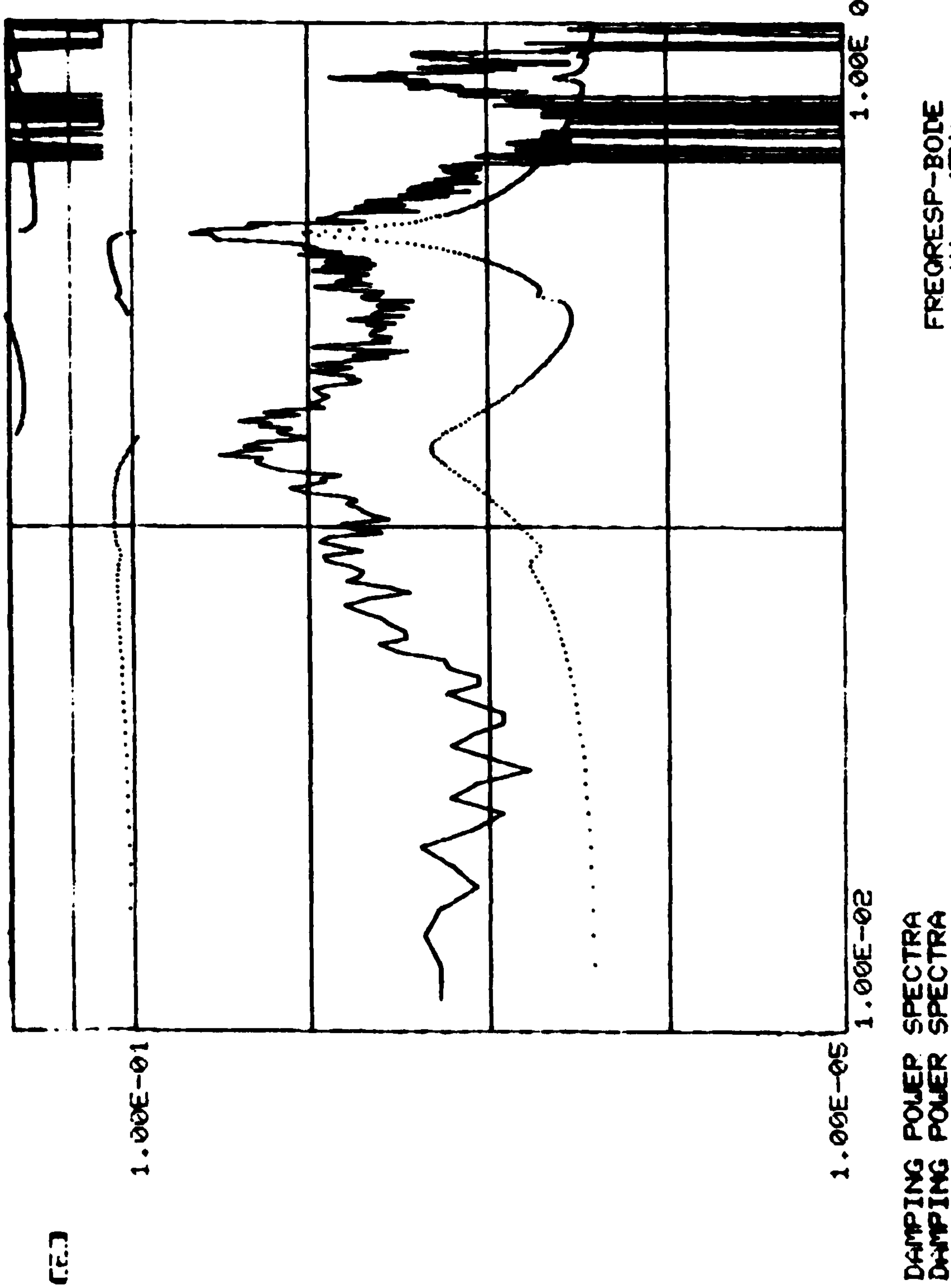
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 46Y+

BLOUGH

ESTIMATED ROOTS (1X+ 47Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.8791E-01	0.5350E-01	0.5376E-06	-0.8084
2	0.1439	0.1106	0.2227E-04	0.1554
3	0.4159	0.7748	0.1626E-10	0.8170
4	0.2859	0.1549E-01	0.4000E-06	1.409
5	0.3823	0.1300E-01	0.3936E-04	-0.3962E-01
6	0.5175	0.2777	0.9353E-06	-2.676
7	0.5577	0.1017	0.1109E-09	-2.539
8	0.6860	0.5633E-01	0.4405E-07	2.781
9	0.7785	0.1137E-01	0.9278E-06	-0.7217E-01
10	0.8656	0.5638E-02	0.1358E-06	-0.1836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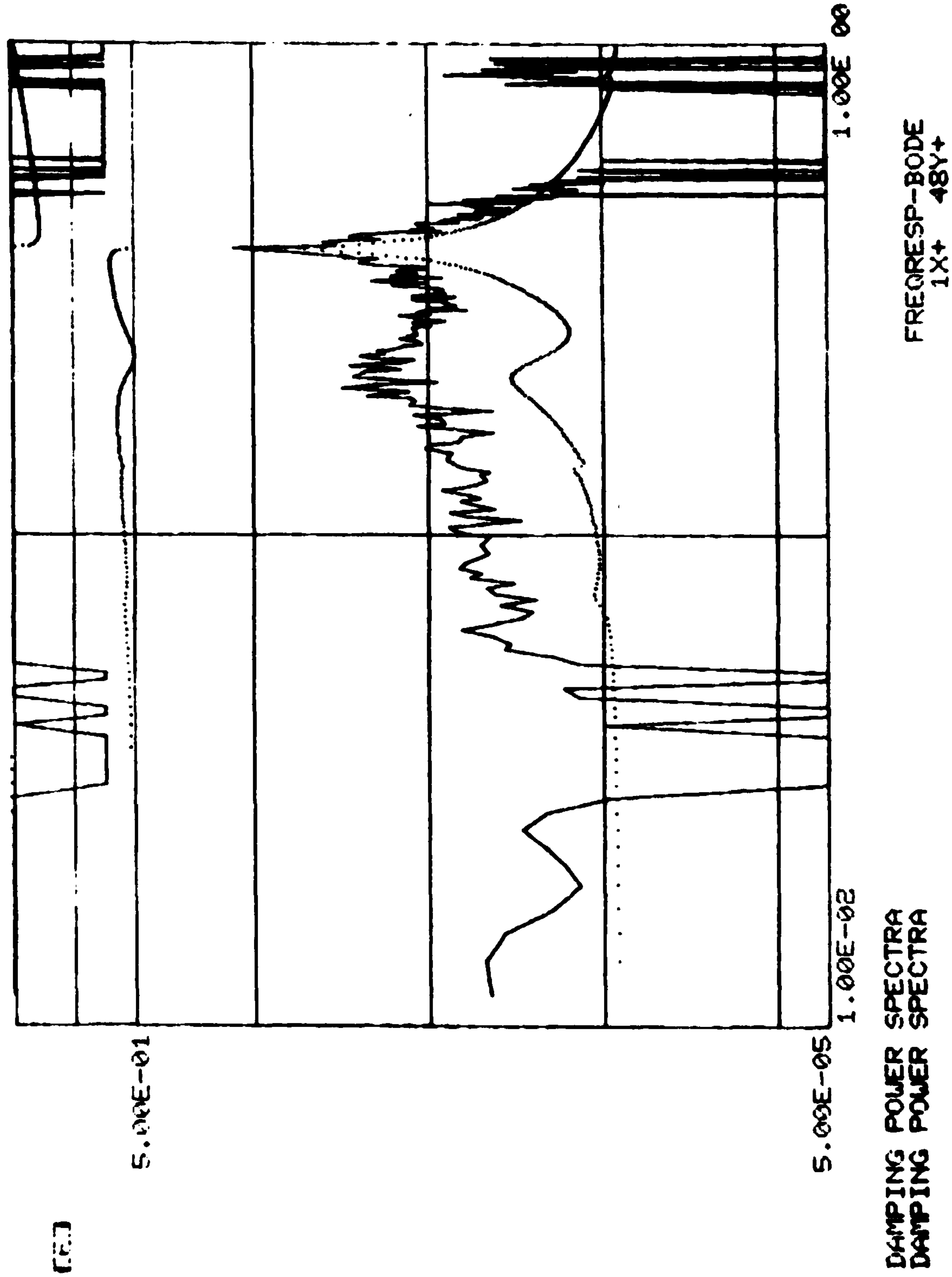
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 47Y+

BLOUGH

ESTIMATED ROOTS (1X+ 48Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.7486E-01	0.7039E-01	0.6694E-06	0.9767
2	0.1382	0.1691E-01	0.4186E-06	-0.7963
3	0.2128	0.9656E-01	0.3470E-04	0.1374
4	0.3349	0.2970E-01	0.1202E-05	-0.8078
5	0.3326	0.1026E-01	0.1630E-03	-0.2193E-01
6	0.4656	0.9585E-01	0.5142E-08	2.433
7	0.4939	0.9422E-01	0.1452E-05	-1.870
8	0.8239	0.1311E-02	0.4281E-07	0.5502
9	0.8660	0.1081E-01	0.6009E-06	0.2004
10	0.9244	0.9656E-02	0.3678E-06	-0.4058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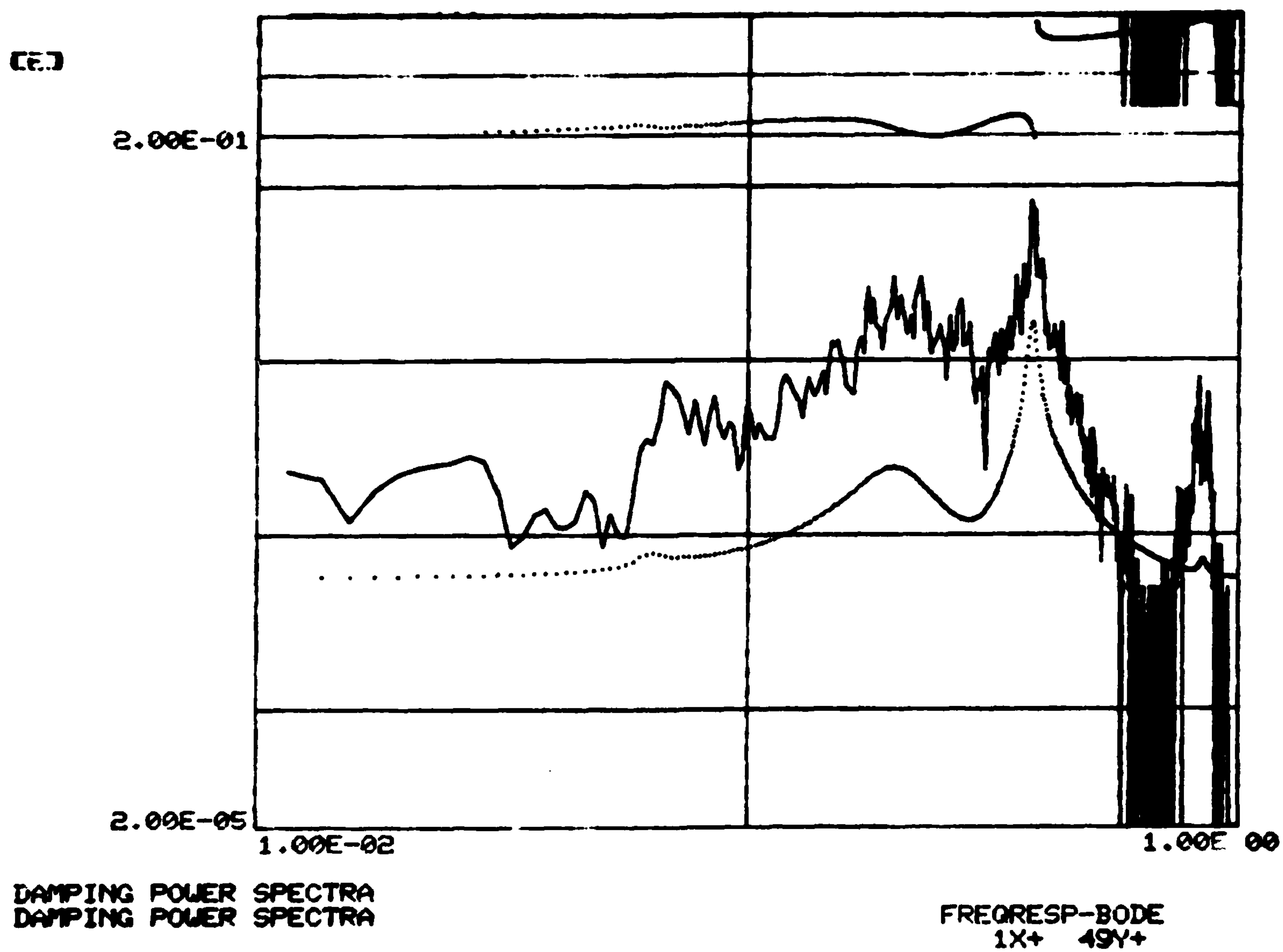
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 48Y+

BLOUGH

ESTIMATED ROOTS (1X+ 49Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.6318E-01	0.8414E-01	0.4959E-06	0.8395
2	0.2003	0.1851	0.4948E-04	0.4115
3	0.2815	0.5503	0.2024E-04	-1.829
4	0.3001	0.7938E-01	0.3870E-06	-2.487
5	0.3802	0.2020E-01	0.9163E-04	-0.1580E-01
6	0.4417	0.6462E-01	0.3840E-05	-1.107
7	0.6757	0.2621	0.1640E-12	0.3023
8	0.8382	0.1717E-01	0.1393E-06	-1.962
9	0.8476	0.1705E-01	0.1523E-05	-0.7943E-01
10	1.012	0.1520	0.8555E-06	-0.3983E-04
11	1.007	0.1161	0.3036E-11	-1.614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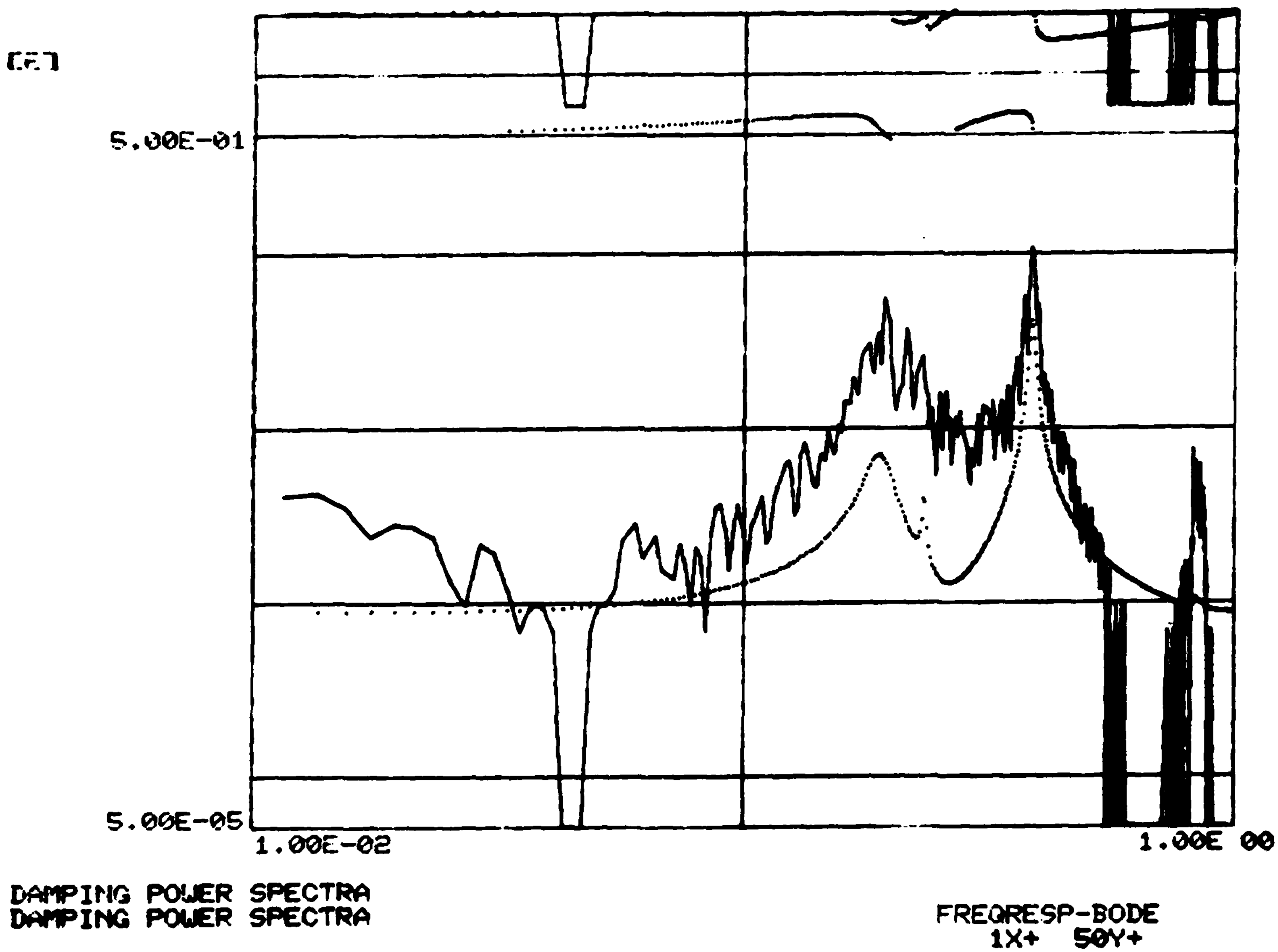
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 49Y+

ESTIMATED ROOTS (1X+ 50Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.5566E-01	1.000	0.6172E-09	-3.110
2	0.1790E-01	1.000	0.4197E-06	0.1116E-03
3	0.1899	0.6598E-01	0.5721E-04	0.1634
4	0.2334	0.9130E-02	0.3931E-05	-0.4235
5	0.3954	0.3266	0.1794E-09	-0.5653
6	0.3866	0.1284E-01	0.1531E-03	-0.3809E-01
7	0.4960	0.5342E-01	0.3024E-07	-2.693
8	0.5048	0.1020	0.1207E-05	-1.787
9	0.7721	0.1838E-01	0.5075E-08	-0.3120
10	0.8324	0.1134E-01	0.9023E-06	0.2495
11	0.8673	0.1024E-01	0.2841E-06	-0.8799



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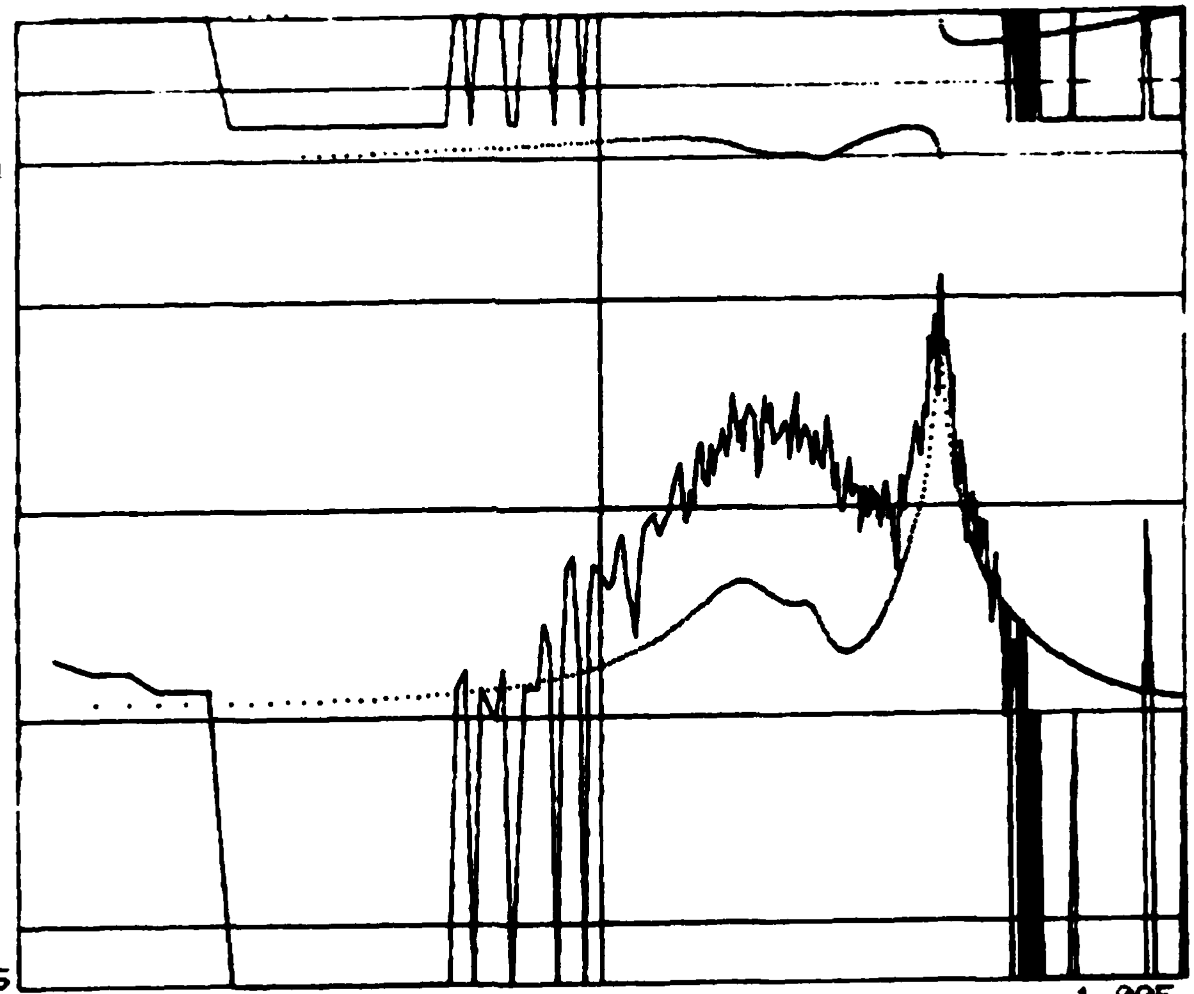
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 50Y+

ESTIMATED ROOTS (1X+ 51Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.9326E-01	1.000	0.5734E-05	-0.1437E-04
2	0.1769	0.1629	0.6121E-04	0.4094
3	0.2306	0.6929E-01	0.1373E-04	-0.3092
4	0.3440	0.1409	0.2866E-04	3.078
5	0.3828	0.1708E-01	0.2429E-03	-0.6260E-01
6	0.3964	0.1237	0.1652E-07	0.3344
7	0.4444	0.3762E-01	0.2061E-06	1.837
8	0.6216	0.4584E-01	0.2248E-07	0.9950
9	0.8667	0.1383E-02	0.4708E-06	0.1861E-01
10	1.002	0.6312E-01	0.1489E-06	-0.2704E-03
11	1.010	0.1413	0.4604E-06	3.141
12	1.004	0.8744E-01	0.2313E-10	-2.965

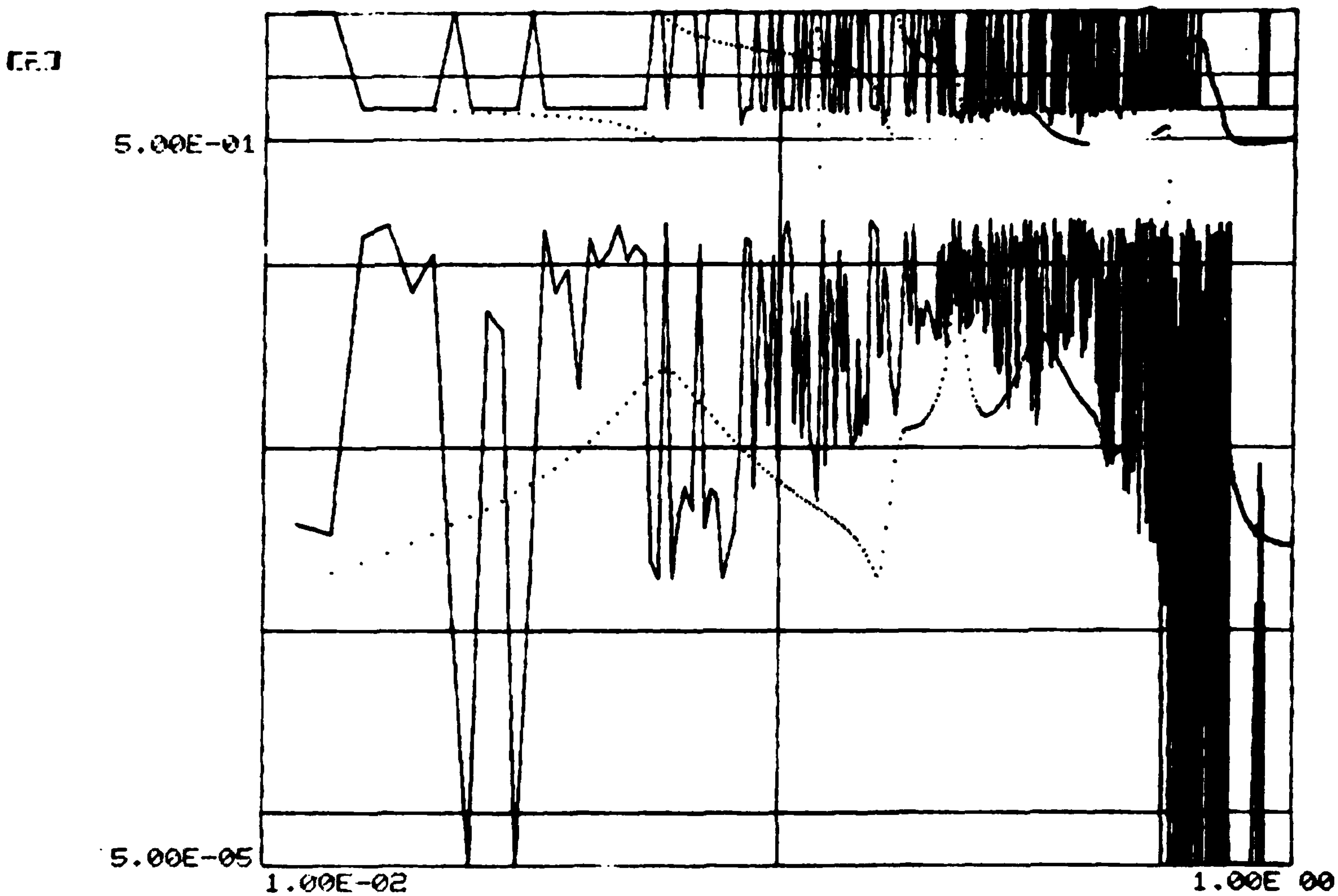


DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 51Y+

ESTIMATED ROOTS (1X+ 52Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.6023E-01	0.1493	0.1973E-03	-0.1165
2	0.1698	0.5164E-01	0.9702E-04	-0.7743E-01
3	0.2211	0.1026E-01	0.2201E-03	2.570
4	0.3187	0.7787E-01	0.7592E-03	1.067
5	0.4273	0.3834E-01	0.6107E-04	-2.519
6	0.5454	0.6034E-01	0.2142E-04	-1.678
7	0.5697	0.2111E-02	0.2889E-03	-0.6666
8	0.6737	0.1665E-01	0.4818E-03	-2.346
9	0.7206	0.2159E-01	0.3702E-03	0.9621
10	0.8531	0.1790E-02	0.5664E-06	0.9221



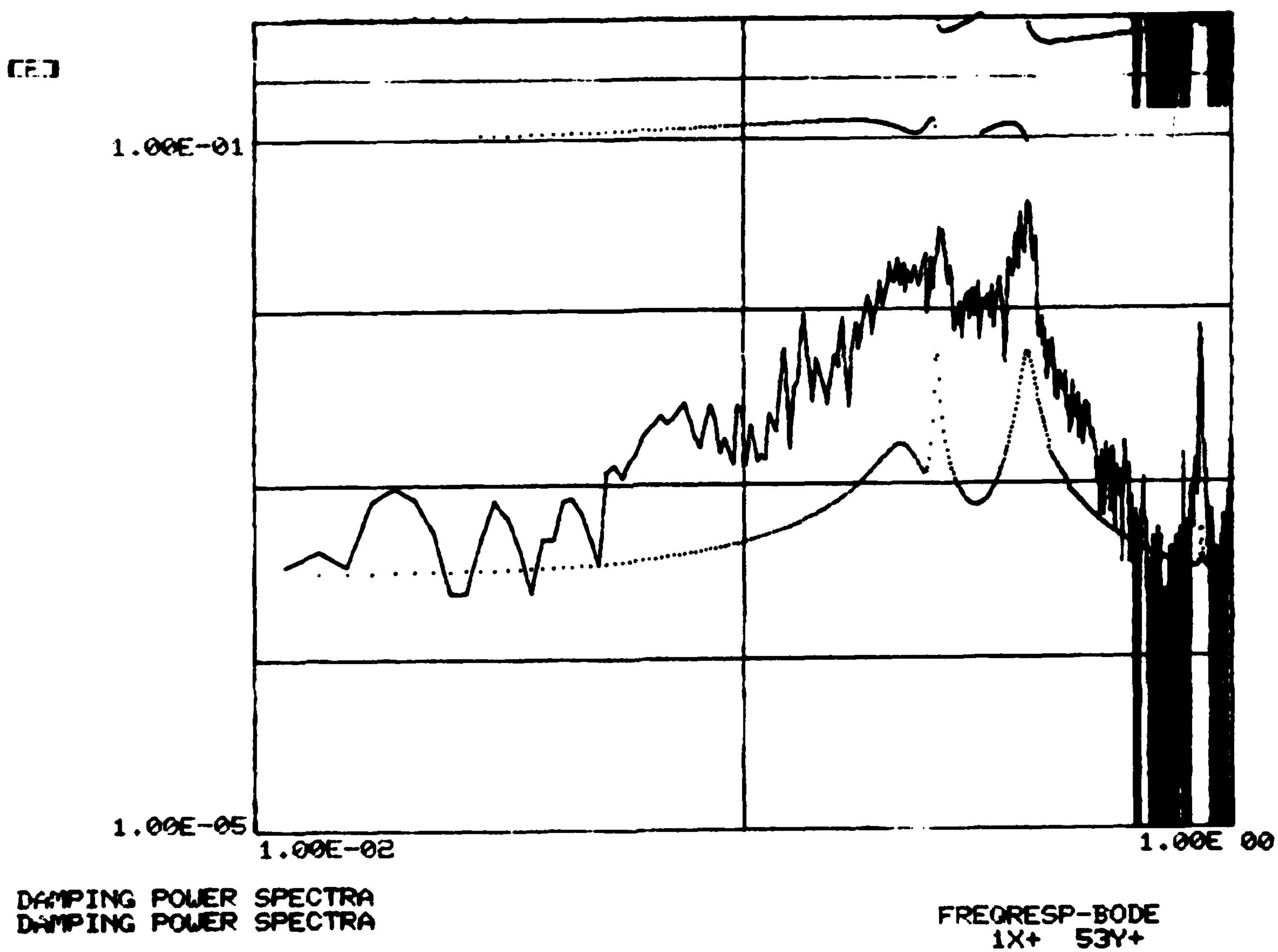
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA

FREQRESP-BODE
1X+ 52Y+

BLOUGH

ESTIMATED ROOTS (1X+ 53Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.1230	1.000	0.2207E-05	-3.142
2	0.1333	0.3457	0.4399E-08	0.1279
3	0.2159	0.1140	0.2362E-04	0.1032
4	0.2490	0.9824E-02	0.9168E-05	0.3926
5	0.3817	0.3035E-01	0.4645E-04	-0.1505
6	0.4237	0.1362E-01	0.9111E-06	2.834
7	0.4639	0.1021E-01	0.2016E-06	2.794
8	0.5938	0.8148E-02	0.1643E-07	0.3943
9	0.7843	0.3484E-01	0.2253E-07	0.9894
10	0.8660	0.3175E-02	0.4683E-06	-0.2234E-01
11	1.000	0.1010E-01	0.6514E-07	-0.8260E-05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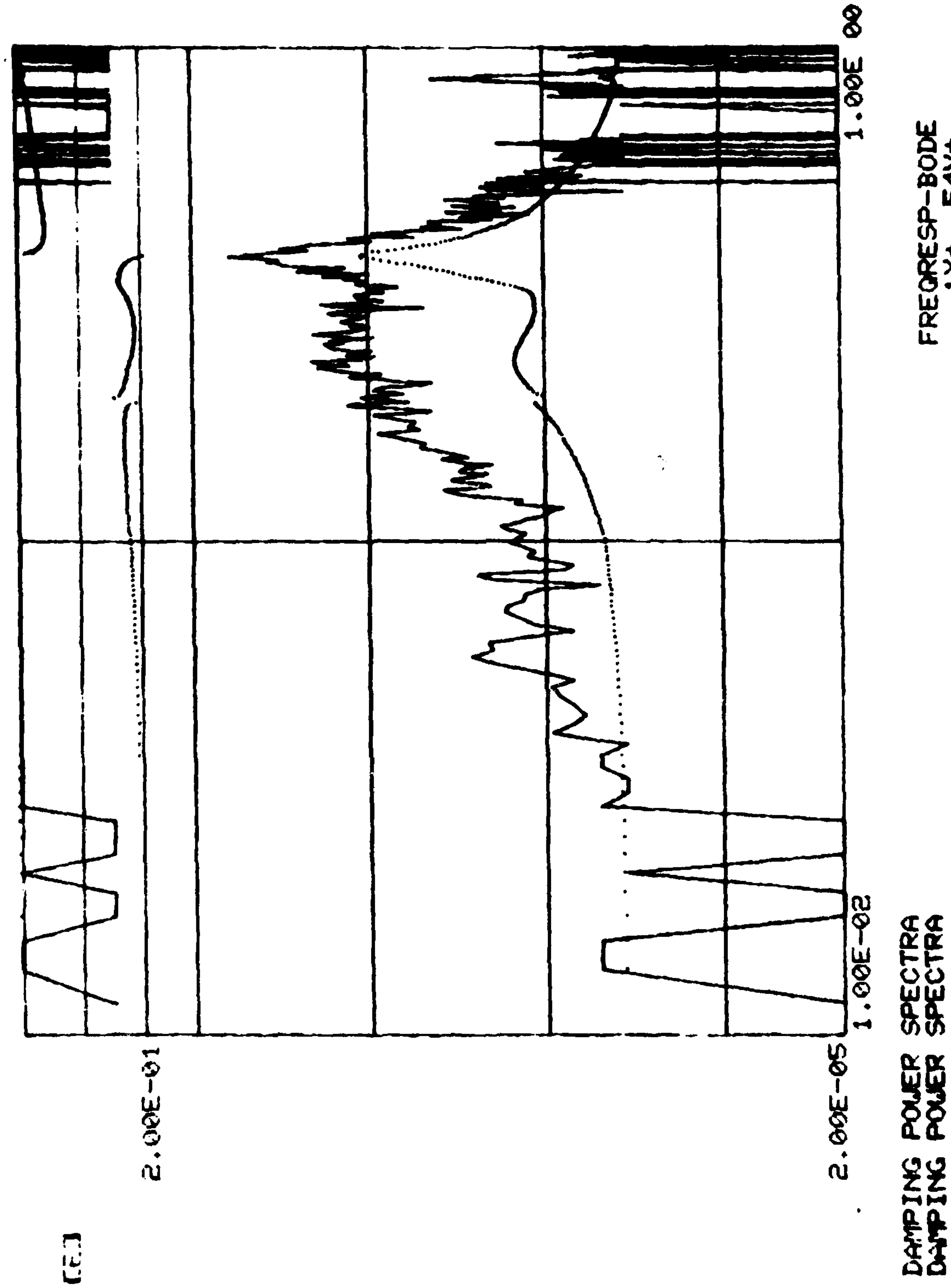
DAMPING POWER SPECTRA
DAMPING POWER SPECTRA
FREQRESP-BODE
1X+ 53Y+

BLOUGH

ESTIMATED ROOTS (1X+ 54Y+)

ROOT	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	0.9232E-01	0.2472	0.3595E-06	-1.344
2	0.1930	0.5762E-02	0.1061E-05	-1.891
3	0.2317	0.1817	0.2779E-04	0.6008
4	0.3337	0.6539E-01	0.1022E-04	-2.886
5	0.3792	0.2480E-01	0.7524E-04	-0.1571
6	0.4645	0.7395E-01	0.1204E-05	1.143
7	0.6288	0.3374E-02	0.1688E-07	0.5421
8	0.6708	0.1689	0.1579E-11	-0.1218
9	0.8677	0.4904E-02	0.2789E-06	-0.6700E-01
10	1.001	0.4452E-01	0.4844E-07	0.7994E-04
11	1.007	0.1136	0.1828E-11	-1.117



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